Estimating the Benefits and Costs of Forming Business Partnerships*

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Abstract

I estimate a matching model of business-partnership formation to quantify the relative importance of productivity gains, financing gains, and the coordination failure of effort provision (moral hazard) among partners. Productivity gains account for 61% of the gain from the observed partnerships. For partners in the first quartile of the wealth distribution, however, financing accounts for 93% of the gain. The cost of moral hazard corresponds to 42% of the entire gain from partnerships. A loan policy specifically targeting partnerships is less effective in improving welfare than a conventional loan policy that provides loans to individual entrepreneurs.

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1 Introduction

Finding a partner is one of the most important decisions for potential start-up owners. Successful business owners often argue that finding the right partner is the key to a firm's success (e.g., Cohen and Eisner [2010]; Kawasaki [2004]). Indeed, many successful companies started as partnerships. Examples include Hewlett-Packard, Procter & Gamble, and Ben & Jerry's. Yet, despite successful examples, only 18% of non-family-owned businesses began as partnerships.[1] Who forms business partnerships, and why? Answering these questions can be valuable not only for potential start-up owners, but also for policymakers who try to boost entrepreneurship via partnerships.

Theoretical studies have identified gains and losses from business partnerships. For example, working with a partner may increase a firm's productivity through a knowledge transfer between partners. Financially constrained entrepreneurs can increase their financing capacity by finding a wealthy partner. The gains, however, can be offset by coordination failure such as moral hazard in teams − the problem of inducing optimal effort by partners when the private return to effort is smaller than the marginal product of effort. The overall impact on entrepreneurs of the option to form a partnership will depend on the magnitude of productivity gains, financing gains, and the cost of moral hazard.

In this article, I quantify the relative importance of productivity gains, financing gains, and the cost of moral hazard in business-partnership formation by developing and estimating a model of partnership formation. The model is an extension of Evans and Jovanovic [1989], an occupational-choice model between a worker and an entrepreneur with a borrowing constraint. I modify their model to incorporate an option to form a partnership.[2] By forming a partnership, financially constrained agents can increase their borrowing capacity. Forming a partnership may also increase productivity. A partnership can be formed if mutual gains in productivity exist or if one partner gains in productivity

[1] The Survey of Income and Program Participation. See section 2 in this article.

[2] To focus on the choice between sole ownership and partnership, I remove the option of becoming a worker.

and the other partner gains in financing despite the cost of moral hazard. Who is matched with whom is determined in a stable matching at which no partner would prefer to be a single owner, and one cannot find two partners who would prefer to form a partnership with each other than to remain with their current partners. The model is estimated by the method of simulated moments using the Survey of Income and Program Participation (SIPP), a nationally representative household-based survey of the US population.

The estimated model implies the increase in productivity accounts for the major gains from partnerships among those who choose to be partners: About 61% of the aggregate gains for all partners are attributed to the productivity gains. However, the gains from financing are the major gains for low-wealth partners. For the partners in the first quartile of the wealth distribution, financing accounts for about 93% of the entire gain from partnerships. The cost of moral hazard is high among the observed partnerships. This cost corresponds to 42% of the entire gain from partnerships.

I use the estimated model to evaluate a policy aimed at boosting entrepreneurship via partnerships. To better understand the effectiveness of such a policy, I first discuss the welfare implication of financial friction when the option to form a partnership is available. In my model, financial friction generates inefficiency through two channels. The first one is from financially constrained single owners who could not invest optimally. In addition to this conventional channel, my model predicts the sorting pattern among start-up owners changes due to financial friction. The welfare loss associated with the former channel is 74.18%, and that with the latter channel is 25.82% of the aggregate welfare losses due to financial friction. The major part of welfare loss associated with changes in the sorting pattern comes from previously single owners who become partners once financial friction is imposed. The welfare loss by this transition group accounts for 24.14% of the aggregate welfare losses due to financial friction.

As a relevant policy simulation, I investigate the impact of a loan policy specifically targeting partnerships. For comparison, I also present the impact of a loan policy that provides loans to individual entrepreneurs. Given the same amount of aggregate loans, the individual-based loan policy is more effective in helping financially constrained single owners increase investment. Moreover, the individual-based loan policy enables partners who would become single owners without financial friction to switch to single ownership, whereas the partnership loan policy does not. Overall, a loan policy specifically targeting partnerships is less effective in improving welfare of start-up owners than a conventional loan policy that provides loans to individual entrepreneurs.

This article is related to the literature on business partnership. Extensive theoretical studies have examined reasons behind business-partnership formation. In the presence of complementarity between partners, a partnership arises as the optimal contract if the knowledge or a tacit human capital between partners is not observable or not contractible (e.g., Teece [1980]; Garicano and Santos [2004]; Morrison and Wilhelm Jr [2004]; Bar-Isaac [2007]). On the other hand, Legros and Newman [1996] show agents' wealth can play an important role in partnership formation in the presence of financial friction. Regarding the cost of partnerships, moral hazard in teams has been the primary theoretical concern since Holmstrom [1982].[3] I contribute to this line of literature by quantifying mechanisms discussed in theoretical studies using a parsimonious equilibrium model of partnership formation. Gaynor and Gertler [1995] and Lang and Gordon [1995] also empirically examine business partnership. They show the extent of moral hazard among partners in professional industries depends on the degree to which the partners spread risk. This article complements their studies by examining different motives for partnership formation that are particularly relevant to start-up firms in general industries.[4]

This article also contributes to the literature on entrepreneurship. Despite the fact that many

[3]The articles that study moral hazard in teams include Radner [1986], Rasmusen [1987], Legros and Matsushima [1991], Legros and Matthews [1993], Miller [1997], Strausz [1999], Battaglini [2006], and Rahman and Obara [2010].

[4]Another difference is that I focus on the formation of business partnerships among start-up owners, whereas Gaynor and Gertler [1995] and Lang and Gordon [1995] examine already established partnerships.

entrepreneurs start a business by forming a partnership, most previous models of entrepreneurship have abstracted such an option. A few exceptions include Basaluzzo [2006] and Ševčík [2015], who quantify to what extent the option to find a partner can help financially constrained agents create a business, and hence increase the aggregate outcome. This article differs from theirs in two important ways. First, Basaluzzo [2006] does not allow mutual gains in productivity, and therefore a partnership is formed only if one partner gains in productivity and the other partner gains in financing. Similarly, Ševčík [2015] focuses on the efficient resource allocation between two separate establishments in the presence of financial friction, and therefore does not allow a possibility that a partnership is formed due to the increase in productivity. By allowing gains in mutual productivity as another source of partnership formation, I show the potential role of financing is small relative to the potential role of productivity gains. Another closely related article is Åstebro and Serrano [2015], who show independent inventors having a partner can significantly increase the probability of commercialization success. Different from Åstebro and Serrano [2015], I allow heterogeneous productivity gains (or losses) and moral hazard across agents induced by a matching model, and use the estimated model to address welfare and policy questions.

The matching model in this article is classified as the imperfectly transferable utility (ITU) model in the matching literature (e.g., Chade et al. [2017]). Despite its relevance, practical applications of the ITU models are rare.[5] I provide one such example that is particularly relevant for start-up formation. Moreover, I quantify the welfare consequences of changes in the sorting pattern among start-up owners due to financial friction, and evaluate the equilibrium consequences of financial subsidies for start-up owners when the option to form a partnership exists.[6]

The article is organized as follows. Section 2 documents some facts about business partnership

[5] An exception is Legros and Newman [2007].

[6] Eeckhout and Munshi [2010] present a model of competitive matching in which potential borrowers and lenders are sorted into groups to exploit gains from trade in the presence of financial friction. They are interested in an informal financial institution in India called Chit funds, whereas I focus on the business-partnership formation.

formation. A model of partnership formation is presented in section 3. Section 4 discusses the identification and estimation of the model. The results are presented in section 5. Section 6 concludes.

2 Facts about Business-Partnership Formation

In this section, I describe the main data set for this study and provide some descriptive patterns related to business partnerships among start-up owners.

Data Construction

I use the SIPP for this study. The SIPP is a nationally representative household-based survey of the US population designed to collect information for income and program participation. I chose the SIPP for several reasons. First, the sample size is large and the survey is nationally representative. The sample size and the time periods of each panel range from approximately 14,000 to 36,000 households and from three to four years, respectively. I use panels including the years 1996, 2001, 2004, and 2008, and hence the time periods span more than 10 years. Thanks to a large number of respondents, I can observe a relatively large number of individuals at the time they become business owners. Second, the SIPP contains information on the respondent's equity share of the business. Using this information, I can define partners versus single owners. Finally, the SIPP provides information on the net worth at the household level, so I can see the relationship between the probability of becoming a partner and household net worth *before* the business is started.

Using the SIPP data, I construct a sample of start-up owners. To limit the influence of labor market participation, I focus on males ages 18 to 65 who are employed before they start their businesses. I drop family businesses because forming a partnership among household members does not increase the total value of household net worth, a mechanism this article investigates.[7]

A business partnership is defined as a group that divides its profit among its members (equity

[7] I present a detailed sample-construction procedure in the web Appendix.

holders).[8] A partner is defined as a business owner whose share of business equity is greater than 1% and less than 99%. Similarly, a business owner whose equity share is greater than or equal to 99% is defined as a single owner.[9]

Data Patterns and Possible Explanations

I now document three data patterns related to those who start a business with a partner, and discuss possible explanations behind those data patterns.

Fact 1. Partners earn significantly more than measurably similar single owners.

Table 1 reports the summary statistics for the incomes of single owners and partners. Figure 1 depicts the density of log incomes for each group. The median income and mean income of partners are greater than those of single owners. To determine whether the same pattern is observed after controlling for other observables, I conduct a regression analysis for log incomes among start-up owners. Table 2 reports the estimates for the regression of log incomes on the partnership dummy and other covariates. I first start with the typical Mincer regression in the regression equation (1) and add various controls for other regressions. For all cases, the partnership dummy is significant and indicates partners earn about 1.65 ($e^{0.5} = 1.65$) times more than single owners.[10]

The fact that the partners earn 1.65 times more than single owners (even after controlling for other observables) may indicate some partners increase their productivity by working with a partner. At the same time, this finding may also reflect the possibility that highly productive individuals are systematically sorted into business partnership even in the absence of productivity gains. I discuss one such case below when I explain Fact 2.

[8]The literature offers a similar definition (e.g., Holmstrom [1982]; Farrell and Scotchmer [1988]; Levin and Tadelis [2005]).

[9]I drop business owners whose equity share is less than or equal to 1%.

[10]If the profit of single owners is greater than that of partners in the later periods, the estimated value of partnerships in this article may be upward biased. In the web Appendix, I show the transfer (failure) rate among partners is greater (smaller) than that of single owners, and this difference becomes insignificant once I control for the initial outcome.

Fact 2. Conditional on starting a business, the predicted probability of becoming a partner is higher in the lower and upper percentile of the wealth distribution than in the middle.

Table 3 shows the summary statistics for the net worth of single owners and partners. Figure 2 depicts the density of net worth for each group. The wealth distribution of partners is more dispersed than that of single owners. To further investigate the relationship between the net worth and the probability of becoming a partner conditional on business ownership, I conduct a probit regression of partnership choice among start-up owners on a fifth-order polynomial in net worth and other covariates. Table 4 reports the estimates from the regression. Figure 3 shows the predicted probability of becoming a partner among start-up owners based on the estimates in Table 4. The predicted probability is evaluated at the sample means of all other controls, except net worth. The predicted probability of becoming a partner is higher in the lower and upper percentile of the wealth distribution than in the middle.

Table 5 reports the number and the proportion of partners for different industries. Partnerships are observed in all industries. Based on the National Survey of Small Business Finances (NSSBF) in 1987, Hurst and Lusardi [2004] categorize construction and services as low-starting-capital industries. They categorize the following as high-starting-capital industries: mining; manufacturing; transportation and public utilities; wholesale and retail trade; and finance, insurance, and real estate. The partnership proportion for high-starting-capital industries is greater than for low-starting-capital industries (p-value 0.027).

The fact that the predicted probability of becoming a partner is higher in the lower percentile of the wealth distribution than in the middle may support the view that start-up owners' financial motives drive partnerships. One aspect of business partnership is similar to equity financing: Financially constrained potential start-up owners may exchange some portion of equity for financing from a partner. Therefore, partnerships can be particularly attractive for low-wealth individuals with a high productivity who are more likely to be financially constrained. The fact that relatively more partners are observed in the high-starting-capital industries may also support this finance-based hypothesis. The low-wealth, high-productivity individuals may form a partnership even in the absence of productivity gains if their partners can increase financing capacity.

Fact 3. Despite the high earnings among the observed partners, only 18% of start-up owners form partnerships.

Table 6 reports the proportion of partners among start-up owners and other statistics for single owners and partners. Excluding family businesses, about 18% of start-up owners choose to form partnerships. The years of education and the years of experience are similar for both types of owners. Partners are more likely to be white. They are also more likely than single owners to be married.

Working as a team can be costly. Extensive theoretical studies identify moral hazard in teams as a primary concern of working with a partner (e.g., Holmstrom [1982]). Hansmann [1996] argues the owners in a partnership often make important decisions, and this decision process can be costly due to heterogeneous interests among the owners. Jensen and Meckling [1992] and Becker and Murphy [1992] note that a firm's productivity can decrease due to difficulties in coordination and communication between partners. These potential costs may explain the fact that only 18% of start-up owners choose to form partnerships, despite the high earnings among the observed partners.

Discussion

Among lawyers and physicians, risk sharing is considered the main motive for joining a partnership (Gaynor and Gertler [1995]; Lang and Gordon [1995]). However, certain features of the data are not consistent with the predictions from the risk-sharing hypothesis. First, most partnerships are formed by two partners.[11] If the motivation behind finding a partner is to share risk, the number of partners

[11] The Kauffman Firm Survey (KFS), a panel study of 4,928 businesses founded in 2004, provides information on the number of owners for a partnership. In the KFS, two-owner partnerships make up 80% of all partnerships.

within a firm would vary depending on the partners' heterogeneous risk preferences. For example, if an agent is highly risk averse, he would find not just one but many partners who have a similar risk preference. Indeed, Gaynor and Gertler [1995] and Lang and Gordon [1995] report sufficient variation in the group size for physicians and law partnerships.[12] Second, as Figure 3 shows, the predicted probability of becoming a partner increases with wealth level after a certain threshold. The opposite relation is predicted if risk sharing is the main motive for forming partnerships and a decreasing absolute risk-aversion preference is assumed.[13]

Although the data pattern is consistent with several hypotheses regarding partnerships, quantifying each one is difficult without further explanation of the data-generating process. In the next section, I present a simple model of partnership formation. I use this model as a measurement tool to quantify the productivity gain, the financing gain, and costs from partnerships. I also use the model for welfare and policy analysis.

3 The Model

Environment

I first describe the model environment. A finite number of agents have decided to start a business. Agent i is characterized by solo productivity $\theta_i \in \mathbb{R}_+$, collaborative skill $g_i \in \mathbb{R}_+$, and net worth $A_i \in \mathbb{R}_+$, which I explain below. Agents have a homogeneous utility function that is linear in consumption, $c_i \in \mathbb{R}_+$,[14] and quadratic in effort, $z_i \in \mathbb{R}_+$: $u(c_i, z_i) = c_i - \kappa\frac{z_i^2}{2}$, where κ is a positive number.

Agents first decide their partners and equity shares (including not to be matched), and then

[12] For example, Gaynor and Gertler [1995] report the average and standard deviation for the number of partners in law partnerships are 21.13 and 8.76, respectively.

[13] Suppose forming a partnership generates a certain payoff, whereas being a single owner generates a risky payoff. Cressy [2000] shows the probability of becoming a single owner increases as individual wealth increases, if agents' preferences exhibit a decreasing absolute risk aversion.

[14] Based on the empirical evidence discussed in section 2, I abstract the risk-sharing motive from the model.

produce the outcome with the pre-determined partner and pre-determined equity share. A shock on the outcome is realized after the production. Who is matched with whom is determined in a stable matching that I define below. A business partnership is formed by no more than two agents.[15]

A financial friction is modeled as a limited commitment between borrowers and lenders. The fact that lenders cannot force borrowers to repay a loan limits the maximum amount of borrowing. Following the literature (e.g., Evans and Jovanovic [1989]), I assume the maximum borrowing amount depends on the borrowers' net worth. More specifically, a single owner can borrow up to $(\lambda - 1)A_i$, where λ is a constant greater than or equal to 1. Therefore, the maximum amount of investment by the single owner is $(\lambda - 1)A_i + A_i = \lambda A_i$. If agent i forms a partnership with another agent j with net worth A_j, the partnership can invest up to $\lambda(A_i + A_j)$.

Before describing the matching market and the type distribution of agents, I first specify single owners' and partners' problem.

Single Owners' Problem

Income as a single owner y_i is determined by: $y_i = \{\theta_i k_i^\alpha z_i^{1-\alpha} - rk_i\}\epsilon_i$, $\alpha \in (0, 1)$, where k_i is capital investment, r is the common risk-free gross interest rate, and ϵ_i is the profit shock for single owners, which is realized after the production.[16] Choice variables are z_i and k_i. Depending on the solo productivity, the income as a single owner will be different even with the same input. I assume the expected value of ϵ_i is equal to 1. The budget constraint implies $c_i = y_i$. A single owner chooses capital investment and effort to solve the following problem:

$$\max_{\{k_i, z_i\}} \mathbb{E}\Big[\big\{\theta_i k_i^\alpha z_i^{1-\alpha} - rk_i\big\}\epsilon_i\Big] - \kappa\frac{z_i^2}{2} \quad \text{subject to} \quad k_i \leq \lambda A_i, \quad k_i > 0. \tag{1}$$

[15] This assumption is based on the fact that a majority of business partnership are formed by two owners. Although allowing a partnership with more than two agents is more realistic, characterizing and simulating the model would be too costly.

[16] Business owners in SIPP were asked to report their profits as business earnings. To make the model consistent with the data, I specified a random shock as a shock to a firm's profit.

Partners' Problem

Consider a partnership formed by agents i and j with pre-determined equity shares ($\tau_i \in (0,1)$ for agent i, $1-\tau_i$ for agent j). The profit of the partnership by agents i and j (y_{ij}) is determined by $y_{ij} = \big\{\theta_{ij} k_{ij}^{\alpha} (z_i + z_j)^{1-\alpha} - r k_{ij}\big\}\epsilon_{ij}$, where θ_{ij} is partnership productivity, k_{ij} is the capital investment by the partnership, and ϵ_{ij} is the profit shock to the partnership, which is realized after the production. The expected value of ϵ_{ij} is equal to 1. Effort in an entrepreneurial team is spent on activities such as generating a business idea, validating the idea, and developing a business model. Effort in these types of activities is hard to observe or monitor. For this reason, I assume effort is not observable. I also assume agents i and j delegate the decision on the capital investment to a financial representative who maximizes the partnership's expected profit. Choice variables for agents i, j, and the financial representative are z_i, z_j, and k_{ij}, respectively. These choice variables are determined as a Nash equilibrium of a simultaneous-move game between agents i, j, and the financial representative. The budget constraint for each agent implies $c_i = \tau_i y_{ij}$ and $c_j = (1-\tau_i) y_{ij}$.

Given $\{z_j, k_{ij}\}$, agent i chooses z_i to solve

$$\max_{z_i} \mathbb{E}\Big[\tau_i \big\{\theta_{ij} k_{ij}^{\alpha} (z_i + z_j)^{1-\alpha} - r k_{ij}\big\}\epsilon_{ij}\Big] - \kappa \frac{z_i^2}{2}. \tag{2}$$

Likewise, given $\{z_i, k_{ij}\}$, agent j chooses z_j to solve

$$\max_{z_j} \mathbb{E}\Big[(1-\tau_i)\big\{\theta_{ij} k_{ij}^{\alpha} (z_i + z_j)^{1-\alpha} - r k_{ij}\big\}\epsilon_{ij}\Big] - \kappa \frac{z_j^2}{2}. \tag{3}$$

Finally, given $\{z_i, z_j\}$, the financial representative chooses k_{ij} to solve

$$\max_{k_{ij}} \mathbb{E}\Big[\big\{\theta_{ij} k_{ij}^{\alpha} (z_i + z_j)^{1-\alpha} - r k_{ij}\big\}\epsilon_{ij}\Big] \quad \text{subject to} \quad k_{ij} \leq \lambda(A_i + A_j), \quad k_{ij} > 0. \tag{4}$$

Matching Market

Now, I describe the matching market and define a stable matching. I first specify how productivity of the partnership by agents i and j is determined. The partnership productivity is a function of the solo productivity of each partner:

$$\theta_{ij} = (g_i\theta_i)^{\frac{1}{2}}(g_j\theta_j)^{\frac{1}{2}}, \tag{5}$$

where $g_i(g_j)$ stands for "collaborative skill" of agent $i(j)$. Knowledge transfer has been considered the key mechanism through which teamwork increases productivity (Lazear [1998]; Argote and Ingram [2000]). However, knowledge transfer is hard and can be costly due to difficulties in coordination and communication (Arrow [1969]; Teece [1977]; Becker and Murphy [1992]; Jensen and Meckling [1992]; Haas and Hansen [2005]; Jones [2009]). Collaborative skill (Hamilton et al. [2003]) and willingness to cooperate (Kosfeld and von Siemens [2011]) are necessary to facilitate the knowledge transfer. More importantly, such characteristics are heterogeneous across individuals independent of the quality of their business idea (θ_i). g_i captures this additional element for teamwork. I call this additional element collaborative skill following Hamilton et al. [2003]. Depending on individual skills for collaborating, it can enhance ($g_i > 1$) or reduce ($g_i < 1$) the solo productivity when partners work together. As a result, productivity as a partner ($g_i\theta_i$) can be different from solo productivity.

With equation (5), if two identical partners are matched, the partnership productivity is the same as their individual productivity as a partner. In particular, if we shut down the collaborative skill by making g_i equal to 1, the partnership productivity is the same as the solo productivity when two identical agents are matched.

Following the merger and acquisition literature (e.g., Rhodes-Kropf and Robinson [2008]; Park [2013]; Xu [2017]), the matching market is modeled as a two-sided market without search and information frictions. More specifically, I assume a market maker randomly selects half the population into one side and the remaining individuals into the other side. Agents are allowed to form a partnership only with an agent on the opposite side.[17]

An equilibrium consists of a **matching** μ, a mapping that specifies which agent on one side is assigned to an agent on the other side (or to become a single owner), and an **equity share** of each partner that satisfies the following two conditions: (1) μ is **stable**, that is, (i) no partner would prefer to be a single owner, and (ii) one cannot find two partners who would both rather form a partnership with each other than remain with their current partners; and (2) the sum of equity shares within a partnership is equal to 1.

Type Distribution

The matching outcome may depend on the distribution over agents' characteristics $\{\theta_i, g_i, A_i\}$.[18] The net worth of each agent is observable in the data, and I use the empirical distribution of A_i when solving the model. I assume θ_i and g_i to be a log-normal distribution following the literature (e.g., Evans and Jovanovic [1989]; Paulson et al. [2006]).

More specifically, the distribution of θ_i is given: $\ln \theta_i = \beta_1 \ln A_i + \beta_2 \ln(edu_i) + \beta_3 \ln(exp_i) + \eta_i$, where $\eta_i \sim N(0, \sigma_\eta^2)$. edu_i and exp_i are the years of education and the potential experience, respectively.[19] In reality, agents with high wealth may invest in different projects than those with low wealth, due to some factors that are not specified in this model. As a result, the return from investment can be different across wealth levels. Explicitly modeling these cases requires additional assumptions, and solving those models is too costly for computation. For this reason, instead of explicitly modeling such cases, I allow the solo productivity to depend on the agent's net wealth. I

[17] The two-sided-market assumption is required to guarantee a stable matching defined below. This method, dividing agents into two sides and matching them as if they came from a two-sided problem, is also suggested by Chade et al. [2017] as a way of guaranteeing the existence of stable matching.

[18] Legros and Newman [2007] characterized a sufficient condition for the economy with ITU to satisfy a positive assortative matching (PAM) regardless of the type distribution. This condition is unlikely to be satisfied in the current setup.

[19] The potential experience is calculated as max$\{$age - years of education - $6, 1\}$.

also allow the solo productivity to be different with education or experience, and normalize the mean of $\ln \theta_i$ conditional on observable variables to be zero.

The distribution of g_i is given: $\ln g_i = g_0 + g_1 \ln(children_i + 1) + g_2 \ln(edu_i) + g_3 \ln(exp_i) + u_i$, where $u_i \sim N(0, \sigma_u^2)$. Education and work experience are inevitably involved with social interaction and therefore may help to develop the collaborative skill. I also allow the collaborative skill to depend on the number of children ($children_i$). The sociology literature has studied the role of the number of siblings on the development of social skills (e.g., Downey et al. [2015]; Yucel and Yuan [2015]). I use the number of children as a proxy for the number of siblings, because research shows the number of children is positively correlated with the number of siblings of each parent (e.g., Murphy and Wang [2001]; Murphy and Knudsen [2002]; Tymicki [2005]).[20]

Characterizing Capital Investment and Effort Choice

I now characterize the amount of capital investment and effort choice by a single owner and by two pre-matched partners with pre-determined equity shares.

I first consider the solution to the single owner's problem. Without the borrowing constraint, the single owner chooses capital investment and effort such that the marginal benefit is equalized with the marginal cost for each input. As a result, the optimal capital investment and effort levels are derived as $k_i^* = \frac{(1-\alpha)}{\kappa}\left(\frac{\alpha}{r}\right)^{\frac{1+\alpha}{1-\alpha}} \theta_i^{\frac{2}{1-\alpha}}$ and $z_i^* = \frac{(1-\alpha)}{\kappa}\left(\frac{\alpha}{r}\right)^{\frac{\alpha}{1-\alpha}} \theta_i^{\frac{1}{1-\alpha}}$, respectively.

When the borrowing constraint is binding (i.e., k_i^* is greater than λA_i), the marginal returns to capital investment are strictly greater than the marginal cost (r). Therefore, a single owner invests up to the borrowing limit λA_i, and choose effort levels so that the marginal returns to effort are equalized with the marginal cost of the effort: $\bar{z}_i = \left(\frac{1-\alpha}{\kappa}\right)^{\frac{1}{1+\alpha}} (\lambda A_i)^{\frac{\alpha}{1+\alpha}} \theta_i^{\frac{1}{1+\alpha}}$.

Let $V(\theta_i, A_i)$ and $y(\theta_i, A_i)$ be the value as a single owner and the income as a single owner for an agent with (θ_i, A_i), respectively. With the optimal capital investment and effort levels derived above,

[20] Note that the parameters for type distributions are jointly estimated with other model parameters.

$V(\theta_i, A_i)$ and $y(\theta_i, A_i)$ are easily characterized.[21]

Next, I consider a partnership formed by agents i and j with pre-determined equity shares. I first consider the case in which the borrowing constraint is not binding. Given $\{z_j, k_{ij}\}$, the best-response function of agent i is determined by solving for the first-order condition of equation (2) with respect to z_i. Likewise, given $\{z_i, k_{ij}\}$, the best-response function of agent j is determined by solving for the first-order condition of equation (3) with respect to z_j. Finally, given $\{z_i, z_j\}$, the best-response function of the financial representative is determined by solving for the first-order condition of equation (4) with respect to k_{ij}. Combining the three best-response functions, the partnership's capital investment and the effort for each partner are derived as $k^*_{ij} = \frac{(1-\alpha)}{\kappa}\left(\frac{\alpha}{r}\right)^{\frac{1+\alpha}{1-\alpha}}\theta_{ij}^{\frac{2}{1-\alpha}}$, $z^*_i = \tau_i\frac{(1-\alpha)}{\kappa}\left(\frac{\alpha}{r}\right)^{\frac{\alpha}{1-\alpha}}\theta_{ij}^{\frac{1}{1-\alpha}}$, and $z^*_j = (1-\tau_i)\frac{(1-\alpha)}{\kappa}\left(\frac{\alpha}{r}\right)^{\frac{\alpha}{1-\alpha}}\theta_{ij}^{\frac{1}{1-\alpha}}$. Each partner's effort increases with his own equity share, but the aggregate effort level $(z_i + z_j)$ is the same regardless of the equity share of each partner. If the borrowing constraint is not binding, the marginal returns to capital investment by a partnership are equalized with the interest rate r.

Now, I consider the case in which the borrowing constraint is binding (i.e., when k^*_{ij} is greater than $\lambda(A_i + A_j)$). To simplify the notation, I denote $A_i + A_j$ as A_{ij}. If k^*_{ij} is greater than λA_{ij}, the partnership's capital investment is λA_{ij} at a Nash equilibrium. Given $\{z_j, k_{ij} = \lambda A_{ij}\}$, the best-response function of agent i is determined by solving for the first-order condition of equation (2) with respect to z_i. Similarly, given $\{z_i, k = \lambda A_{ij}\}$, the best-response function of agent j is determined by solving for the first-order condition of equation (3) with respect to z_j. Combining the best-response functions of agent i and j, the effort for each partner is derived as $\bar{z}_i = \tau_i\left(\frac{1-\alpha}{\kappa}\right)^{\frac{1}{1+\alpha}}(\lambda A_{ij})^{\frac{\alpha}{1+\alpha}}\theta_{ij}^{\frac{1}{1+\alpha}}$ and $\bar{z}_j = (1-\tau_i)\left(\frac{1-\alpha}{\kappa}\right)^{\frac{1}{1+\alpha}}(\lambda A_{ij})^{\frac{\alpha}{1+\alpha}}\theta_{ij}^{\frac{1}{1+\alpha}}$.

Let $V_{ij}(\theta_{ij}, A_{ij}, \tau_i)$ and $y_{ij}(\theta_{ij}, A_{ij}, \tau_i)$ be the value and income from partnership for agent i who is matched with agent j, respectively. Similarly, let $V_{ji}(\theta_{ij}, A_{ij}, 1-\tau_i)$ and $y_{ji}(\theta_{ij}, A_{ij}, 1-\tau_i)$ be the

[21] I present a detailed derivation in the web Appendix.

value and income from partnership for agent j who is matched with agent i, respectively. With the optimal effort by agent i and j and the optimal capital investment by the financial representative, characterizing the value and income from partnership for each agent is straightforward.[22]

Planner's Solution

Before discussing an equilibrium sorting pattern, I characterize benefits and costs, relative to being a single owner, of a partnership. To do so, I present a planner's problem in which the planner still faces financial friction, but can mitigate the coordination failure of effort provision by partners. The planner solves the following problem:

$$\max_{\{z_i, z_j, k_{ij}\}} \mathbb{E}\Big[\big\{\theta_{ij} k_{ij}^{\alpha}(z_i+z_j)^{1-\alpha} - r k_{ij}\big\}\epsilon_{ij}\Big] - \frac{\kappa}{2}\big(z_i^2 + z_j^2\big) \quad \text{subject to} \quad k_{ij} \leq \lambda A_{ij}, \quad k_{ij} > 0.$$

When the borrowing constraint is not binding, the first-order conditions with respect to z_i, z_j, and k_{ij} are

$$(1-\alpha)\theta_{ij} k_{ij}^{\alpha}(z_i+z_j)^{-\alpha} = \kappa z_i, \tag{6}$$

$$(1-\alpha)\theta_{ij} k_{ij}^{\alpha}(z_i+z_j)^{-\alpha} = \kappa z_j, \tag{7}$$

$$\alpha\theta_{ij} k_{ij}^{\alpha-1}(z_i+z_j)^{1-\alpha} = r. \tag{8}$$

The first-order conditions imply $\hat{z}_i = \hat{z}_j = \hat{z}_{ij} = \left(\frac{1-\alpha}{\kappa}\right)\left(\frac{\alpha}{r}\right)^{\frac{\alpha}{1-\alpha}} \theta_{ij}^{\frac{1}{1-\alpha}}$ and $\hat{k}_{ij} = 2\left(\frac{1-\alpha}{\kappa}\right)\left(\frac{\alpha}{r}\right)^{\frac{1+\alpha}{1-\alpha}} \theta_{ij}^{\frac{2}{1-\alpha}}$. If $\hat{k}_{ij}$ is greater than λA_{ij}, the planner's capital investment is constrained at λA_{ij}, and $\tilde{z}_i = \tilde{z}_j = \tilde{z}_{ij} = \left(\frac{1}{2}\right)^{\frac{\alpha}{1+\alpha}} \left(\frac{1-\alpha}{\kappa}\right)^{\frac{1}{1+\alpha}} (\lambda A_{ij})^{\frac{\alpha}{1+\alpha}} \theta_{ij}^{\frac{1}{1+\alpha}}$.

The key difference between this planner's problem and the partners' problem specified above can be explained by the first-order conditions of each problem. For example, the first-order condition with respect to z_i without the borrowing constraint is $(1-\alpha)\left(\frac{k_{ij}}{z_i+z_j}\right)^{\alpha}\theta_{ij} = \kappa z_i$ for the planner's

[22] A detailed derivation is shown in the web Appendix.

problem, and it is $\tau_i(1-\alpha)\left(\frac{k_{ij}}{z_i+z_j}\right)^{\alpha}\theta_{ij} = \kappa z_i$ for the partners' problem. Although the marginal cost of effort is the same, the marginal benefit of effort is smaller in the partners' problem than in the planner's problem, because the marginal product of effort is shared according to the pre-determined equity share in the partners' problem. As a result, the aggregate effort and investment level are always lower in the partners' problem than in the planner's problem.

Suppose the planner divides the output equally to each partner. Then, the values from the planner's problem for agents i and j are equal. That is, $V^E_{ij}(\theta_{ij}, A_{ij}) = V^E_{ji}(\theta_{ij}, A_{ij}) = \Big(\frac{1}{2}\{\theta_{ij}\hat{k}^{\alpha}_{ij}(2\hat{z}_{ij})^{1-\alpha} - r\hat{k}_{ij}\} - \frac{\kappa}{2}\hat{z}^2_{ij}\Big) \cdot (1 - I^E_{ij}) + \Big(\frac{1}{2}\{\theta_{ij}(\lambda A_{ij})^{\alpha}(2\tilde{z}_{ij})^{1-\alpha} - r\lambda A_{ij}\} - \frac{\kappa}{2}\tilde{z}^2_{ij}\Big) \cdot I^E_{ij}$, where I^E_{ij} is an indicator function whose value is 1 if $\hat{k}_{ij} > \lambda A_{ij}$, and zero otherwise. The superscript E indicates the value is from the planner's problem.

Consider agent i with solo productivity θ_i and net worth A_i. With the value function from the planner's problem in hand, it is straightforward to prove the following propositions.

Proposition 1. $V^E_{ij}(\theta_i, 2A_i) = V(\theta_i, A_i)$

Suppose Agent i forms a partnership with an identical individual, and the solo and partnership productivity are the same. Proposition 1 says the sum of each partner's value as a single owner is equal to the aggregate value of the partnership. Three channels make the value from partnership different from the value from single ownership.

Proposition 2. $V^E_{ij}(\theta_{ij}, 2A_i) = V(\theta_{ij}, A_i) > V(\theta_i, A_i)$ *if and only if* $\theta_{ij} > \theta_i$

The graphical representation of Proposition 2 is shown in Figure 4 (a). A point (θ_i, θ_{ij}) in the figure represents a situation in which agent i with solo productivity θ_i is matched with agent j (with the same net worth) so that the partnership productivity is θ_{ij}. The white (black) color indicates agent i's value from the planner's problem is greater (smaller) than his value from single ownership. Suppose two partners have the same net worth. The value from the planner's problem is greater than the value from single ownership if and only if the partnership productivity is greater than the solo productivity. The value difference in Proposition 2 captures the value generated by the partnership productivity.

Proposition 3. *$V_{ij}^{E}(\theta_i, A_{ij}) > V(\theta_i, A_i)$ if and only if $A_j > A_i$ and $\lambda A_i < k_i^*$*

The graphical representation of Proposition 3 is shown in Figure 4 (b). A point (A_i, A_j) in the figure represents a situation in which agent i with net worth A_i is matched with agent j with net worth A_j (and with $\theta_i = \theta_{ij}$). The white (black) color indicates agent i's value from the planner's problem is greater (smaller) than his value from single ownership. The gray color indicates agent i's value from the planner's problem is the same as his value from single ownership. Although the partnership productivity and the solo productivity are the same, if an agent is financially constrained as a single owner and his partner's net worth is greater than his own net worth, the value from the partnership is strictly greater than the value from single ownership if the coordination failure of effort provision by partners does not exist. The value difference in Proposition 3 captures the value generated by financing. Notice that if no financial friction exists, the value difference induced by financing is zero.

Proposition 4. *$V_{ij}(\theta_{ij}, A_{ij}, \tau_i) + V_{ji}(\theta_{ij}, A_{ij}, 1 - \tau_i) < 2V_{ij}^{E}(\theta_{ij}, A_{ij})$*

Finally, with the given productivity and net worth, the aggregate value from the partnership problem is strictly less than that from the planner's problem, due to less effort provision in the partnership problem. Following Holmstrom [1982], I call the value difference in Proposition 4 the cost of moral hazard.[23]

To summarize, for a partnership formed by agents i and j, the value difference between partnership

[23] Among many potential costs from partnerships, I choose to model the moral hazard in teams as an explicit cost of partnerships, because many theoretical studies have considered moral hazard a primary concern of working with a partner. Moreover, implementing moral hazard in the current setup is straightforward and easy.

and single ownership can be decomposed as

$$
\begin{aligned}
&V_{ij}(\theta_{ij}, A_{ij}, \tau_i) - V(\theta_i, A_i) + V_{ji}(\theta_{ij}, A_{ij}, 1-\tau_i) - V(\theta_j, A_j) \\
&= \underbrace{V_{ij}^E(\theta_{ij}, 2A_i) - V(\theta_i, A_i) + V_{ij}^E(\theta_{ij}, 2A_j) - V(\theta_j, A_j)}_{\Omega_1} \\
&+ \underbrace{V_{ij}^E(\theta_{ij}, A_{ij}) - V(\theta_{ij}, A_i) + V_{ij}^E(\theta_{ij}, A_{ij}) - V(\theta_{ij}, A_j)}_{\Omega_2} \\
&+ \underbrace{V_{ij}(\theta_{ij}, A_{ij}, \tau_i) + V_{ji}(\theta_{ij}, A_{ij}, 1-\tau_i) - 2V_{ij}^E(\theta_{ij}, A_{ij})}_{\Omega_3}, \qquad (9)
\end{aligned}
$$

where Ω_1 represents productivity gains, Ω_2 represents financing gains, Ω_3 captures the cost of moral hazard.

Equilibrium and Sorting Pattern

I now explain how I solve a stable matching. Due to the non-linear effort in the utility function and the borrowing constraint, the utility possibility frontier for two partners is not linear. In the matching literature, this type of environment is called the imperfectly transferable utility (ITU) matching model (see, e.g., Legros and Newman [2007] and Chade et al. [2017]). The existence of equilibria in this class of economies has been established by Crawford and Knoer [1981] and Kaneko [1982]. To find an equilibrium, I apply a transfer-adjustment process proposed by Crawford and Knoer [1981].[24]

To see how financial friction affects the sorting pattern across individuals, I parameterize an economy as below and simulate 2,000 individuals:

$$\{\kappa, \alpha\} = \{1, 0.3\}, \quad \ln\theta_i \sim N(0,1), \quad \ln g_i \sim N(1,1), \quad \ln A_i \sim N(0,10).$$

I first find a stable matching without financial friction. The sorting pattern with respect to log ability as a partner, $\ln(g_i\theta_i)$, is shown in the upper panel of Figure 5. The positive assortative matching is

[24] A detailed procedure is explained in Appendix A.

clearly observed, and the correlation between two partners' log ability is 0.998. The sorting pattern with respect to log wealth is shown in the upper panel of Figure 6. If no financial friction exists, wealth does not affect a match between two partners, and as a result, we observe no relationship between two partners' wealth levels. The correlation between two partners' log wealth is near zero.

With the same simulated individuals, I next solve the stable matching by assuming λ is equal to 1. Before discussing the sorting pattern, I first show how the proportion of single owners and partners changes. As Panel A of Table 7 shows, the number of partners increases about 2 percentage points when the borrowing constraint is imposed. When financial friction is introduced, additional gains from forming a partnership arise due to the gains from financing, as described in Proposition 2. As a result, the total number of partners increases.

Financial friction changes not only the number of partners, but also the sorting pattern among partners. The sorting pattern with respect to log ability as a partner is shown in the lower panel of Figure 5. Compared to the sorting pattern without financial friction, the extent of positive assortative matching, especially among high-ability partners, is reduced. The correlation between two partners' log ability becomes 0.904, as opposed to near 1 in the case without financial friction.

The sorting pattern with respect to log wealth is shown in the lower panel of Figure 6. Contrary to the case without financial friction in which the correlation between two partners' log wealth is near zero, a clear negative assortative matching pattern is observed. When $\lambda = 1$, the correlation between two partners' log wealth is -0.46.

Panel B of Table 7 shows several moments of $\ln(\text{income}) \times \ln(\text{wealth})$ for partners before and after financial friction is imposed. Once financial friction is introduced, changes in the shape of the distribution − especially mean and skewness − are clearly observed. The financial friction changes the sorting pattern among partners, in particular with respect to wealth, which in turn changes the shape of the earnings distribution of partners conditional on wealth.

To summarize, financial friction may increase the number of partners, decrease the extent of positive assortative matching with respect to one's ability as a partner, and induce a negative assortative matching with respect to wealth. Financial friction also changes the shape of the earnings distribution of partners conditional on wealth. In section 5, I quantify the welfare implication of these changes due to financial friction.

4 Identification and Estimation

In this section, I explain how the model is identified and estimated.

Earning Shocks

Before discussing the identification, I first specify the earnings shocks. As Figure 1 shows, the distribution of conditional income for business owners looks similar to a log-normal distribution with a possible realization of a zero or a negative income. To capture this stylized feature of the data, I assume the earning shocks as follows:

$$\epsilon_i = \tilde{\epsilon}_i - P_s, \quad \ln \tilde{\epsilon}_i \sim N(\mu_s, \sigma_s^2), \quad \mathbb{E}[\epsilon_i] = 1, \quad P_s \text{ is a positive constant,}$$

$$\epsilon_{ij} = \tilde{\epsilon}_{ij} - P_p, \quad \ln \tilde{\epsilon}_{ij} \sim N(\mu_p, \sigma_p^2), \quad \mathbb{E}[\epsilon_{ij}] = 1, \quad P_p \text{ is a positive constant.}$$

Identification

The model consists of the joint distribution of solo productivity and productivity as a partner, the matching function, and parameters for production/utility functions and for the borrowing constraint. I first explain how certain types of data limitations require identifying restrictions on my theory, and then provide an informal argument for how available data moments can identify the model parameters under these identifying restrictions.

Because we do not observe the alternative outcome (e.g., an outcome as a partner for those who choose to become single owners) or the match between partners, nonparametrically identifying the joint distribution of solo productivity and productivity as a partner is not feasible. To identify the joint distribution, I impose a parametric assumption (joint log-normal distribution) and the assumption that productivity as a partner is proportional to solo productivity (θ_i) depending on the individual's collaborative skill (g_i). I also normalize the mean of $\ln \theta_i$ to be zero conditional on observable variables.

Without knowing who is matched with whom, especially with respect to their productivity, identifying the matching function is hard. Therefore, I impose the functional-form assumption specified in equation (5).

The cost of moral hazard specified in Proposition 4 crucially depends on parameters for production/utility functions. Without the information on the actual capital expenditure, separately identifying the production parameter (α) from the effort parameter (κ) is difficult. Therefore, I use the estimate from Evans and Jovanovic [1989], and set $\alpha = 0.22$.

Although we can always find a stable matching for a given set of parameters via the transfer-adjustment process,[25] the uniqueness of such stable matching is typically not guaranteed. In this sense, I assume the observed outcome in the data is the outcome of the stable matching found via the transfer-adjustment process.[26]

I now explain how available data moments can identify the model parameters. Given the assumption of lognormality, the distribution of the solo productivity is characterized by the mean parameters ($\{\beta_1, \beta_2, \beta_3\}$) and the standard deviation (σ_η). Also, the distribution of the collaborative skill is characterized by the mean parameters ($\{g_0, g_1, g_2, g_3\}$) and the standard deviation (σ_u). I first explain the moments that identify the solo productivity. Suppose β_1 has increased. The mean of log income

[25]The stable matching typically changes depending on which side of the market first proposes the transfer. However, in the current setup, the two sides become identical as the number of agents increases, and therefore, the stable matching rarely changes regardless of which side first offers the transfer.

[26]We can guarantee a unique stable matching if we impose the equal-division restriction on the transfer between a match. In the previous version of this article, I imposed such a restriction, and found the equilibrium sorting pattern is similar to the one in the current version.

by single owners conditional on net worth will increase in the stable matching. Therefore, the mean of log income by single owners conditional on net worth can be used to identify β_1. Similarly, β_2 and β_3 can be identified by the mean of log incomes by single owners conditional on education and experience. Note that observed log incomes by single owners reflect a truncated distribution of the solo productivity. As σ_η increases, the unconditional mean of log income by single owners will increase. Therefore, the unconditional mean of log income by single owners will be informative to identify σ_η.

Given the parameters for solo productivity, several other moments can help identify the distribution of collaborative skill. Suppose g_0 has increased. More people can generate a high value as a partner, and the number of partners will increase in the stable matching. As a consequence, the proportion of partners among start-up owners will increase. Therefore, the proportion of partners among start-up owners can identify g_0. Likewise, the proportion of partners among start-up owners conditional on education and on experience identify g_1 and g_2, respectively. Similar to the identification argument for σ_η, the unconditional mean of log income by partners can identify σ_u.

Given the distribution of the solo productivity and the collaborative skill, the distribution of log income by partners changes as the effort parameter (κ) changes. Therefore, the mean of log income by partners conditional on observable variables can be used to identify κ.

The following moments can be useful to identify λ. First, the proportion of partners among business owners conditional on net worth is affected by λ given β_1. Consider, for example, financially constrained single owners with a small net worth. As λ decreases, the value of those constrained single owners also decreases. As a result, those owners whose value as a single owner is marginally higher than their value as a partner will change their choice to a partner, and the proportion of partners among business owners conditional on the small net worth will increase. Second, as shown in section 3, λ also affects the shape of the distribution − especially mean and skewness − of partners' income conditional on net worth. Therefore, these moments for log income by partners conditional on net worth can help identify λ.

Finally, the higher moments of log income by single owners and the proportion of zero- or negative-income single owners identify σ_s and P_s. Similarly, the higher moments of log income by partners and the proportion of zero- or negative-income partners identify σ_p and P_p.

Estimation

The model is estimated by the method of simulated moments:

$$\hat{\psi} = \arg\min_{\psi} [\mathbf{m}_{\text{sim}}(\psi) - \mathbf{m}_{\text{data}}]' \mathbf{W} [\mathbf{m}_{\text{sim}}(\psi) - \mathbf{m}_{\text{data}}],$$

where $\mathbf{m}_{\text{data}}$ is the M $\times$ 1 vector of empirical moments, $\mathbf{m}_{\text{sim}}(\psi)$ is the the M $\times$ 1 vector of simulated moments given a set of parameters ψ, and $\mathbf{W}$ is the M $\times$ M weighting matrix.

To estimate the model, a stable matching is solved via the transfer-adjustment process for each simulation. The moments choice is guided by the identification argument. In particular, I use the proportion of partners and moments for business earnings conditional on observable variables. I choose the weighting matrix to be a diagonal matrix that contains the inverse of variances of the data moments. The number of parameters and the number of moments are 15 and 19, respectively.

The variance-covariance matrix of the estimator is given by $(\mathbf{D}'\mathbf{W}\mathbf{D})^{-1}\mathbf{D}'\mathbf{W}\Sigma\mathbf{W}\mathbf{D}(\mathbf{D}'\mathbf{W}\mathbf{D})^{-1}$, where $\mathbf{W}$ is the 19 $\times$ 19 weighting matrix, $\mathbf{D} = \frac{\partial \mathbf{m}_{sim}(\psi)}{\partial \psi}|_{\psi=\hat{\psi}}$ is the 19 $\times$ 15 derivative matrix of the moment vector with respect to the model parameters evaluated at the estimates, and Σ is a 19 $\times$ 19 matrix, which I calculate following Gourieroux and Monfort [1996].

Log value of net worth is required for estimation, and I replace negative net worth with \$1. Before estimation, I normalize one unit of net worth and income as \$10,000 in 2011.[27] The gross risk-free interest rate, r, is assumed to be 1.1 following the literature (e.g., Evans and Jovanovic [1989]; Xu [1998]; Paulson et al. [2006]).

[27] Evans and Jovanovic [1989] used \$1,000 in 1976 as one unit.

The full set of simulated and targeted moments is shown in Table 8. The model fit is reasonably good. Note that I do not target the distribution of equity shares in the data for estimation. At the same time, the model generates the distribution of equity share as an equilibrium outcome. As another model-validity check, I compare the histogram and kernel density for simulated equity shares with those from data. I simulate the model 100 times with the estimated parameters, and present all the equity shares from those simulations. For expositional purposes, I replace an equity share below one half with 1 minus the equity share. Figure 7 shows the result. A majority of equity shares are concentrated around one half, and although small, various other equity shares are observed in the data. A similar pattern is observed in the simulated equity shares. Even though a high concentration around one half is not observed, a majority of equity shares are between 50% and 60%. Various other equity shares are observed, but those equity shares represent a relatively small proportion.[28]

Given that λ is the key parameter determining the financing gain, I present the sensitivity of λ with respect to the moment vectors used for estimation following Andrews et al. [2017]. The sensitivity of the parameter estimates with respect to the moment vector is a 15×19 matrix $\Lambda = -(\mathbf{D}^{'}\mathbf{W}\mathbf{D})^{-1}\mathbf{D}^{'}\mathbf{W}$.[29] The sensitivity of the borrowing constraint parameter λ is the row of the sensitivity matrix corresponding to λ. The result is shown in Figure 8. Each point in the figure represents a local measure of the relationship between λ and the corresponding moment of the data used for estimation. The results are in line with the identification argument. In particular, the skewness of partners' income conditional on net worth is most sensitive to λ.

[28]The fact that equity shares from data are highly concentrated around one half could be due to factors which are not modeled in this article. For example, Hellmann and Wasserman [2016] argue inequality aversion by partners can generate an equal split of equity among partners although equal splitting is inefficient.

[29]Given the scales of the moments are not always comparable, I multiply the mth column of Λ by the standard deviation of the mth moment following Gayle and Shephard [2019].

5 Results

In this section, I present the results of this article. I first discuss the parameter estimates, and then quantify the gains and losses from partnership. I also discuss the welfare implication of financial friction, and evaluate the effectiveness of government loans tied to partnerships.

Estimates

Table 9 shows the estimates for the model parameters. The borrowing-constraint parameter (λ) is estimated at 1.6089, suggesting agents can invest up to about 1.6 times their net worth. This result supports the existence of borrowing constraints for start-up owners, which is consistent with the previous findings in the literature.

The relationship between net worth and solo productivity (β_1) is estimated to be very small and insignificant. This result is different from Evans and Jovanovic [1989], who find a significant and negative relationship between net worth and entrepreneurial productivity. Their model does not take into account the fact that an individual who could start a business with a partner despite a very low wealth level is more likely to have a very high ability as a partner. As a result, their model may absorb this additional mechanism as a relationship between net worth and entrepreneurial productivity.

Education and experience positively influence both solo productivity and collaborative skill. For example, a 10% increase in the years of education leads to approximately a 2.4% increase in solo productivity. Likewise, a 10% increase in the years of education leads to an approximate 3.1% increase in collaborative skill. The coefficient for the number of children, which I use as a proxy for the number of siblings, is positive but not significant.

The constant term for log value of the collaborative skill is estimated at -1.1567, suggesting a large productivity *loss*, on average, from working with a partner compared to working alone. Note that I explicitly control for the cost of moral hazard in business partnerships. The cost of moral hazard is not enough to explain the fact that only about 18% of agents choose partnerships conditional on business ownership. This finding supports the view that the cost of knowledge transfer outweighs the benefit for most individuals.[30]

Decomposing Aggregate Gains

With the estimates of structural parameters, I now quantify the net gains from observed partnerships relative to the total income generated by those partnerships. To do so, I simulate the model economy and calculate the net gains ($\Omega_1+\Omega_2+\Omega_3$ in equation (9)) for every partnership and then sum them up. I divide this aggregate net gain by the income generated by all partnerships. After 200 simulations, I report the average value.[31] The resulting estimate is 0.36 with the standard error being 0.008.[32] This finding implies the aggregate net gain from partnerships is 36% of the aggregate income generated by all partnerships.

Next, I decompose the aggregate net gain from partnerships with respect to productivity gains (Ω_1), financing gains (Ω_2), and moral hazard (Ω_3) based on equation (9). This decomposition exercise would tell us the relative importance of productivity gains, financing gains, and the cost of moral hazard in explaining the observed business partnerships. In doing so, I simulate the model economy and calculate each component (Ω_1, Ω_2, or Ω_3) for every partnership and then sum them up. I normalize each summed value by the gross gains ($\Omega_1 + \Omega_2$) from all partnerships.

The result is shown in the first row of Table 10. A larger part of aggregate gains − before subtracting the losses from moral hazard − are explained by gains from the partnership productivity: 60.88% of the aggregate gains are explained by the partnership productivity. The remaining 39.12% is explained by the gains from financing. The cost of moral hazard among partners is estimated as

[30] Merges between firms are often thought to be driven by the gains in productivity. The low estimated g is also in line with the fact that most mergers − even *conditional on being merged* − are not successful and are often being divested (see, for example, Banal-Estañol and Seldeslachts [2011]).

[31] For all counterfactual simulations, I simulated the economy 200 times and report the average value.

[32] I use a bootstrap method to calculate standard errors for counterfactual analyses.

42.29% of the aggregate gains from partnerships.[33]

Figure 9 shows the scatter plot for productivity gains, financing gains, and the cost of moral hazard for each partnership. With the estimated model parameters, I simulate the model economy 50 times and present the results for all the partnerships from the simulations.[34] Although many partnerships enjoy both productivity and financing gains, a considerable amount of partnerships are formed solely due to mutual productivity gains. The opportunity cost of coordination failure is higher as the partnership productivity becomes higher, as reflected by a higher cost of moral hazard.

To investigate the variation between productivity and financing gains across the wealth distribution, I conduct the decomposition exercise conditional on those whose net wealth is below and above the 25th percentile of the wealth distribution. The second and third rows of Table 10 present the results. Financing gains are the major benefit for the low-wealth partners: 92.7% of the aggregate gains for the low-wealth partners is explained by financing. By contrast, for high-wealth partners, almost all of the gains (98.85%) are generated by the partnership productivity. Note the relationship between solo productivity and wealth is estimated to be small. As a result, most of the financially constrained agents are concentrated in the lower percentile of the wealth distribution. The gains from financing are positive only for those financially constrained agents. For them, financing gains explain most of the gains from partnership.

The cost of moral hazard could be potentially higher if many partnerships cannot be formed due to the moral hazard. To check this possibility, I simulate a counterfactual economy in which every partner commits to provide the effort in the planner's problem specified in section 3. I compare the proportion of partners in this counterfactual economy with the estimated (benchmark) economy. The result is shown in the second row of Table 11. Without the moral hazard, the proportion of partners

[33] Other frictions besides the one specified in my model may also generate a suboptimal effort choice by partners. Therefore, the cost of moral hazard estimated here may also contain the cost of coordination failure by other frictions.

[34] For illustration purposes, I present $\{\Omega_1, \Omega_2, -\Omega_3\}$ for each partnership.

would have been 32.78% $-$ about 14 percentage points higher than the observed partners. This result implies the cost of moral hazard is high both in the intensive and the extensive margin.[35]

Welfare Analysis

The estimated model can be used to evaluate a loan policy aimed at boosting entrepreneurship via partnerships. Before evaluating a policy, I first discuss the welfare implication of financial friction when the option to form a partnership is available. Understanding the types of inefficiency discussed in this section can be helpful to understand the effectiveness of policies presented in the following section. Note that unlike most of the previous models with financial friction, the current model features an unexplored distortion due to financial friction: changes in the sorting pattern due to financial friction, as illustrated in section 3.

I first report the change in the proportion of partners due to financial friction in Table 11. Compared to the estimated economy (benchmark), the proportion of partners is smaller if financial friction does not exist. This finding implies mitigating financial friction could be an important motive for partnership formation.

To understand the welfare consequences of changes in partnership proportion and the sorting pattern among partners, in Table 12 (a), I decompose the welfare losses for each transition group before and after financial friction is imposed: (i) single owner to single owner, (ii) single owner to partner, (iii) partner to single owner, and (iv) partner to partner. I first normalize the aggregate welfare losses due to financial friction as 100: $\sum_{i=1}^{N}(W_i - W_i^f)\mathbb{I}_{\text{Loss}} = 100$. W_i^f and W_i refer to the value of agent i with and without financial friction, respectively. $\mathbb{I}_{\text{Loss}}$ represents the indicator function for $W_i - W_i^f$ being positive.

A large part of the welfare losses comes from single owners who remain single owners but with

[35] The loss from the moral hazard relative to the outcome without the moral hazard is also substantial. I divide the aggregate income in the estimated economy by the aggregate income without the moral hazard. The resulting estimate is 0.83 with the standard error being 0.01, suggesting the aggregate income reduces by 17% due to the moral hazard.

constrained borrowing. The welfare loss from this group accounts for 74.18% of the aggregate welfare losses due to financial friction. Before further discussion, I define the following three situations as mismatches driven by financial friction: a single owner become a partner, a partner becomes a single owner, and a partner changes his partner.

The welfare loss from the mismatch accounts for 25.82% (24.14 + 0.02 + 1.66) of the aggregate welfare losses due to financial friction. Of this percentage, single owners who become partners due to a financial motive explain a major part of welfare losses (24.14%). Partners who become single owners explain 0.02%. The remaining 1.66% is due to partners who remain partners but are matched with less productive agents than the partners with whom they would have been matched if no financial friction were present.

Financial friction generates welfare *gains* for a certain group of people. Table 12 (b) decomposes the welfare gains for each transition group. Some single owners who become partners benefit from financial friction. The magnitude of their gains corresponds to 0.71% of the aggregate welfare losses due to financial friction. Some partners also benefit from financial friction. The magnitude of the welfare gains for these partners corresponds to 2.55% of the aggregate welfare losses due to financial friction. These gains are generated by some partners and single owners with high wealth who have an opportunity to find more productive agents than the partners with whom they would have been matched if no financial friction were present.

In Table 13, I further investigate welfare losses and gains due to the mismatch with respect to the wealth level. Most welfare losses due to the mismatch are generated by individuals whose net worth is less than the 25th percentile of the wealth distribution. These welfare losses account for 96.77% of the entire welfare loss from the mismatch. This finding is in sharp contrast to the fact that only 12.25% of the gains from the mismatch are generated by low-wealth individuals.

Policy Experiments

I now investigate the impact of a loan policy specifically targeting business partnerships. For comparison, I also present the impact of an individual-based loan policy provided that the total amount of government loans is the same across two policies.

The Small Business Technology Transfer (STTR) program sponsored by the US government provides funds only for partnerships between small businesses and nonprofit research institutions. The goal is to promote commercialization of innovations in science by encouraging partnership between the public and private sector. Although the firms targeted by the STTR program may be quite different from the firms in the sample for this article, the policy experiment in this section can be helpful to understand an equilibrium impact of the STTR program as well as other loan policies specifically targeting partnerships.

Loans Targeted for Business Partnerships

I first experiment with a loan policy in which a government commits to lend financially constrained business partnerships up to \$25,000 at the market interest rate r. I choose this loan amount mainly because it matches the average small-business-loan amount guaranteed by the U.S. Small Business Administration.[36] This loan amount is worth about 18% of the median of partners' assets, and about 64% of the median of partners' first-year business income.

Table 14 (a) shows a change in the proportion of partners among all start-up owners. Compared to the estimated economy, the proportion of partners increases about six percentage points (from 18.92% to 24.76%). The left column of Table 15 (a) shows the welfare gains by the partnership loans for each transition group. About 15.4% $(11.38 - 0.64 + 4.66)$ of the aggregate welfare losses from financial friction disappear. Major gains are coming from those who were previously single owners

[36] In the second quarter of 2015, the average amount of small business loans is about \$25,000 (Source: Office-Of-Advocacy [2015]).

and switch to partners. By providing a subsidy tied to partnership, financially constrained single owners have an additional incentive to form a partnership, and some do so.

The partnership loan can also alleviate the partner mismatch. In doing so, some partners who can be matched with more productive partners due to financial friction are worse off due to the loan policy. The value of their wealth in the presence of financial friction reduces once other partners can access the loans provided by the government.

Individual-based Loans

To compare the impact of the partnership loan with a more conventional small business subsidy, I experiment with a loan program in which the government commits to lend $25,000 to *some* financially constrained individuals as single owners at the market interest rate. To make the policy comparable to the partnership loan, I make only those individuals who are financially constrained as single owners eligible for this loan program. I allow those individuals who receive the loan to find a partner with this government loan. I make the number of individuals who receive the loan the same as the number of partnerships that benefit from the partnership loan policy. In this way, the total government loan amount becomes the same across the two policies. I *randomly* allocate the loans to financially constrained individuals as single owners.[37] I call this policy the small business loan policy.

Table 14 (a) shows a change in the proportion of partners among all start-up owners. In contrast to the partnership loan policy, the proportion of partners decreases about one percentage point (from 18.92% to 17.71%), compared to the benchmark. The right column of Table 15 (a) shows the welfare gains by the small business loan for each transition group. About 23.83% $(17.60 + 0.2 + 5.17 + 0.86)$ of the aggregate welfare losses from financial friction disappear, which is much larger than the welfare gains from the partnership loan (15.4%). A major gain comes from financially constrained single

[37]As I explain below, the main finding from two policy analyses is that the individual-based government loan program is better than the loan program targeted for business partnerships. The random-allocation rule provides a conservative estimate for the difference between the two policies in terms of efficiency.

owners who can increase investments thanks to the small business loan.

Interestingly, the gains from the other transition groups are also considerable for the small business loan. Almost all of such gains come from individuals who switch from being partners to being single owners after the policy is implemented. Under the partnership loan, partners who would have become single owners without financial friction still remain as partners. By contrast, under the small business loan, these partners become single owners and generate welfare gains.

To check the extent to which the potential benefits of relaxing the borrowing constraint depend on the amount of loan, I increase the loan per business from $25,000 to $50,000. The results are summarized in Table 14 (b) and Table 15 (b). Compared to the $25,000 loan per business, the proportion of partners rarely changes, and the total welfare gains marginally increase in both policies. The reason is that a major part of welfare losses from financial friction come from a small number of single owners whose investment is largely constrained. The optimal amount of investment by these highly productive single owners is so large that the increase in the loan amount from $25,000 to $50,000 rarely mitigates the friction they face.

Overall, the individual-based loan program is more effective than the loan program targeted only for business partnerships. The former helps constrained single owners invest more, and helps partners who would have been single owners without financial friction become single owners.

A caveat of the policy analyses in this section is that I did not take into account applicants' default decisions. Endogenizing agents' default decisions can be valuable for analyzing small business loan programs but is outside the scope of this article. Another abstraction is that I did not allow the possibility that constrained business owners could be discouraged and become workers. Depending on this additional extensive margin, the predicted number of partners would be different from the prediction in this section.

6 Conclusion

I empirically investigate the formation of partnerships among start-up owners. I first document that partners earn significantly more than single owners. In addition, relatively more partners are observed in the lower and upper percentile of the wealth distribution than in the middle. Despite the high earnings among observed partners, the proportion of partners is much smaller than the proportion of single owners. To further investigate the decision to form a partnership, I develop and estimate a matching model based on Evans and Jovanovic [1989].

Using the estimated model, I first show productivity gains explain a large portion of the gains from the observed partnerships. Financing gains are the major benefit to partners with low net worth. Financial friction generates inefficiency by altering the sorting pattern among partners. A loan policy specifically targeting partnerships is less effective in improving welfare than a conventional loan policy that provides loans to individual entrepreneurs.

Tables and Figures

Table 1: Income

	Single owners	Partners
Obs.	829	186
Mean	43,085	65,038
10%	3,362	8,805
50%	22,980	38,841
90%	100,137	138,228

NOTE: This table shows the summary statistics for incomes of single owners and partners given that they reported a positive income. USD in 2011.

Table 2: Income Regression for Business Owners

	(1)	(2)	(3)	(4)
VARIABLES	ln(Income)	ln(Income)	ln(Income)	ln(Income)
Partnership	0.616	0.540	0.528	0.472
	(0.112)	(0.109)	(0.108)	(0.108)
Education	0.0861	0.0370	0.0326	0.0325
	(0.0160)	(0.0168)	(0.0180)	(0.0178)
Experience	0.0284	0.0134	0.0144	0.00694
	(0.0143)	(0.0141)	(0.0140)	(0.0143)
Experience^2	-0.000630	-0.000385	-0.000415	-0.000256
	(0.000315)	(0.000309)	(0.000308)	(0.000310)
Previous income		0.00583	0.00545	0.00490
		(0.000784)	(0.000780)	(0.000774)
Net worth		0.000204	0.000179	0.000215
		(0.000102)	(0.000101)	(0.000101)
Industry dummy	No	No	Yes	Yes
Other dummies	No	No	No	Yes
Observations	1,015	1,015	1,015	1,015
R-squared	0.061	0.118	0.147	0.180

NOTE: This table shows the estimates for the regression of log incomes on partnership dummy, years of education, years of experience and its square, previous income, net worth, and industry and other dummies. Partnership means the partnership dummy. Previous income indicates the wage income before starting a business. Previous income and net worth are normalized by 1,000 USD in 2011. Other dummies include race, marital status, and year dummies. Standard errors are in parentheses.

Table 3: Net Worth

	Single owners	Partners
Obs.	941	204
Mean	243,195	328,427
10%	-7,263	-5,463
50%	90,706	138,591
90%	697,136	910,628

NOTE: This table shows the summary statistics for net worth of single owners and partners. USD in 2011.

Table 4: Net Worth and Partnership Choice among Start-up Owners

VARIABLES	Partnership dummy
Net worth/100,000	-0.0666
	(0.0543)
$(\text{Net worth}/100{,}000)^2$	0.0487
	(0.0163)
$(\text{Net worth}/100{,}000)^3$	-0.00607
	(0.00184)
$(\text{Net worth}/100{,}000)^4$	0.000258
	(8.12e-05)
$(\text{Net worth}/100{,}000)^5$	-3.47e-06
	(1.21e-06)
Education	-0.0168
	(0.0181)
Experience	-0.0115
	(0.00436)
Previous income	0.000940
	(0.000786)
Race	0.309
	(0.150)
Married	0.335
	(0.104)
Constant	-1.095
	(0.301)
Observations	1,145
Pseudo R-squared	0.0443

NOTE: This table reports the estimates for a probit regression of becoming a partner among start-up owners. Previous income is normalized by 1,000 USD in 2011. Race is the dummy variable for white. Standard errors are in parentheses. The coefficients for wealth variables are jointly significant (p-value 0.0034).

Table 5: Industry Decomposition

Industry	Single owners	Partners	Partners (%)
Agriculture, forestry, fishing, and hunting	56	18	24
Construction	260	49	16
Manufacturing	30	10	25
Wholesale trade	30	5	14
Retail trade	64	22	26
Transportation, warehousing, and utilities	48	4	8
Information	16	1	6
Finance, insurance, real estate, and rental and leasing	50	20	29
Professional, scientific, management, administrative, and waste management	204	39	16
Services	183	36	16
High-starting-capital industry	222	61	22
Low-starting-capital industry	443	85	16

NOTE: This table reports the number and proportion of partners for different industries. Services include the following: Business and repair services; Personal services; Educational, health and social service; Arts, entertainment, recreation, accommodations, and food service; Public administration; and other services. High-starting-capital industry includes Manufacturing; Wholesale and Retail trade; Transportation, warehousing, and utilities; and Finance, insurance, real estate, and rental and leasing. Low-starting-capital industry includes Construction and Services.

Table 6: Proportion of Partners, and Other Statistics for Single Owners and Partners

	Single owners	Partners
Obs.	941	204
Proportion (%)	82	18
Experience (Year)	20.48	19.68
Education (Year)	14.04	14.35
Race (White)(%)	85.26	91.44
Married (%)	61.59	71.66

NOTE: This table reports the proportion of partners among start-up owners, and other statistics for single owners and partners. Mean is reported unless otherwise indicated.

Table 7: Financial Friction and Sorting Pattern

	Economy w.o Financial Friction	Economy with $\lambda = 1$
	[Panel A]	
Proportion of partners	0.83	0.85
Corr($\ln(g_i\theta_i)$, $\ln(g_j\theta_j)$)	0.998	0.904
Corr($\ln A_i$, $\ln A_j$)	0.045	-0.46
	[Panel B]	
ln(earning)$\times$ ln(wealth)		
Mean	-0.132	1.258
Std.	12.959	12.696
Skewness	-0.035	0.563
Kurtosis	9.78	9.614

NOTE: This table reports the proportion of partners and sorting patterns from two simulated economies. I simulate 2,000 individuals from the economy characterized as follows: $\{\kappa, \alpha\} = \{1, 0.3\}$, $\ln \theta_i \sim N(0,1)$, $\ln g_i \sim N(1,1)$, $\ln A_i \sim N(0,10)$. The first column reports the results without financial friction, and the second column reports the results with $\lambda = 1$.

Table 8: Data vs. Simluated Moments

#	K_i	Z_i	Data (m_{data}) $\frac{1}{n}\sum_{i=1}^{n} K_i Z_i$	Simulated (m_{sim}) $\frac{1}{n}\sum_{i=1}^{n} \tilde{K}_i(\psi) Z_i$
1	$\mathbb{I}_{pi}$	1	0.1782	0.1817
2		A_i	6.0354	4.2219
3		Education$_i$	2.5293	2.6772
4		Experience$_i$	3.7362	3.9380
5		Children$_i$	0.2393	0.1948
6	$\pi_{si}\ \mathbb{I}_{si}^{\text{pos.}}$	1	0.4761	0.4468
7		A_i	17.0180	20.2199
8		Education$_i$	7.1280	6.5670
9		Experience$_i$	10.3319	11.4204
10	$\left\{\pi_{si} - \text{ave.}(\pi_{si})\right\}^2 \mathbb{I}_{si}^{\text{pos.}}$	1	1.0531	2.6194
11	$\mathbb{I}_{si}^{\text{neg.}}$	1	0.0978	0.1092
12	$\pi_{pi}\ \mathbb{I}_{pi}^{\text{pos.}}$	1	0.2074	0.2134
13		A_i	8.9612	6.7836
14		Education$_i$	3.0762	3.2319
15		Experience$_i$	4.4029	5.0388
16		Children$_i$	0.2590	0.2366
17	$\left\{\pi_{pi} - \text{ave.}(\pi_{pi})\right\}^2 \mathbb{I}_{pi}^{\text{pos.}}$	1	0.2191	0.3406
18	$\left\{\left(\pi_{pi}A_i - \text{ave.}(\pi_{pi}A_i)\right)/\text{std.}(\pi_{pi}A_i)\right\}^3 \mathbb{I}_{pi}^{\text{pos.}}$	1	0.6331	0.4507
19	$\mathbb{I}_{pi}^{\text{neg.}}$	1	0.0157	0.0140

NOTE: This table compares the actual and simulated moments. $\mathbb{I}_{si}$ is the indicator function for single owners. $\mathbb{I}_{pi}$ is the indicator functions for partners. A_i is net worth (10,000 USD in 2011). Education is the years of education, Experience is the years of experience, and children is the number of children. π_{si} is log income conditional on agent i being a single owner, and agent i reports a positive income. $\mathbb{I}_{si}^{\text{pos.}}$ is the indicator function for single owners who report a positive income. $\mathbb{I}_{si}^{\text{neg.}}$ is the indicator function for single owners who report a negative or zero income. π_{pi} is log income conditional on agent i being a partner and agent i reporting a positive income. $\mathbb{I}_{pi}^{\text{pos.}}$ is the indicator function for partners who report a positive income. $\mathbb{I}_{pi}^{\text{neg.}}$ is the indicator function for partners who report a negative or zero income.

Table 9: Estimates for the Model Parameters

Parameters	Variables	Estimates	Std. errors
	Panel A: solo productivity		
β_1	ln(Net worth)	1.6e-5	(0.0031)
β_2	ln(Education)	0.2415	(0.0195)
β_3	ln(Experience)	0.0387	(0.0058)
σ_η	Std. of η	0.1343	(0.0041)
	Panel B: collaborative skill		
g_0	Constant	-1.1567	(0.0343)
g_1	ln(Children+1)	0.0001	(0.0027)
g_2	ln(Education)	0.3126	(0.0050)
g_3	ln(Experience)	0.0221	(0.0033)
σ_u	Std. of u	0.3215	(0.0081)
	Panel C: other parameters		
λ	Collateral constraint	1.6809	(0.2561)
κ	Preference	0.2084	(0.0707)
	Panel D: earnings shocks		
σ_s	Std. of $\ln \tilde{\epsilon}_s$	1.5053	(0.0428)
P_s	–	0.0630	(0.0018)
σ_p	Std. of $\ln \tilde{\epsilon}_p$	1.0223	(0.0823)
P_p	–	0.1661	(0.0135)

NOTE: This table presents the estimates for the model parameters. Asymptotic standard errors are in parentheses.

Table 10: Decomposing Benefits and Costs of Partnerships

	Productivity gain (%)	Financing gain (%)	Moral hazard (%)
All partners	60.88	39.12	-42.29
	(2.32)	(2.32)	(0.48)
Partners < 25th wealth	7.30	92.70	-37.20
	(1.38)	(1.38)	(0.62)
Partners $\geq$ 25th wealth	98.85	1.15	-45.90
	(0.28)	(0.28)	(0.57)

NOTE: The first row shows the decomposition of benefits and costs from partnerships for all partners. The aggregate gains for all partners (productivity gain + financing gain) are normalized as 100. The second and third row show the decomposition of benefits and costs from partnerships below and above the 25th percentile of the wealth distribution, respectively. The aggregate gains for each group are normalized as 100. Bootstrap standard errors are in parentheses.

Table 11: Moral Hazard, Financial Friction, and Proportion of Partners

	Partners (%)
Benchmark	18.92
	(1.38)
Economy w.o Moral hazard	32.78
	(1.57)
Economy w.o Financial Friction	12.35
	(1.26)

NOTE: This table compares the proportion of partners from the estimated (benchmark) economy with (1) those from the estimated economy but removing moral hazard, and (2) those from the estimated economy but removing financial friction. Bootstrap standard errors are in parentheses.

Table 12: Decomposing Welfare Losses and Gains Due to Financial Friction by Each Transition Group (the numbers are normalized by the aggregate welfare losses due to financial friction)

	After Financial Friction			
	(A)		(B)	
	Welfare Loss		Welfare Gain	
	Single owners	Partners	Single owners	Partners
Single owners	74.18	24.14	0	-0.71
	(1.51)	(1.35)	-	(0.03)
Partners	0.02	1.66	0	-2.55
	(0.004)	(0.19)	-	(0.24)

NOTE: Column A of this table presents welfare losses with respect to each transition group after financial friction is introduced. Column B of this table presents welfare gains for each transition group due to financial friction. The aggregate welfare losses from financial friction are normalized as 100: $\sum_{i=1}^{N}(W_i - W_i^f)\mathbb{I}_{\text{Loss}} = 100$. W_i^f and W_i represent the value of agent i with and without financial friction, respectively. $\mathbb{I}_{\text{Loss}}$ represents the indicator function for $W_i - W_i^f$ being positive. Note the numbers are normalized by the aggregate welfare losses, so the numbers for welfare gains are presented as negative numbers. Bootstrap standard errors are in parentheses.

Table 13: Welfare Losses and Gains Due to the Mismatch by Low-Wealth Individuals (the numbers are normalized by the aggregate welfare losses due to financial friction)

	Welfare Loss	Welfare Gain
All agents (a)	25.82	-3.26
	(1.51)	(0.25)
Agents < 25th wealth (b)	24.99	-0.40
	(1.40)	(0.08)
(b/a)×100	96.77	12.25
	(0.32)	(1.64)

NOTE: This table presents losses and gains due to the mismatch (specified in section 5) by low-wealth individuals. Agents < 25th wealth indicates individuals whose net worth is less than the 25th percentile of the wealth distribution. The numbers are normalized by the aggregate welfare losses due to financial friction. Bootstrap standard errors are in parentheses.

Table 14: Proportion of Partners after Two Policy Experiments

Loan per Business	Type of Policies	Partners (%)
	Benchmark	18.92
		(1.38)
(a) $25,000	Partnership loan	24.76
		(1.21)
	Small business loan	17.71
		(1.34)
(b) $50,000	Partnership loan	24.75
		(1.21)
	Small business loan	17.65
		(1.33)

NOTE: This table compares the proportion of partners after two policies described in section 5 are implemented. The benchmark economy is the estimated economy. Bootstrap standard errors are in parentheses.

Table 15: Decomposing Welfare Gains by Two Policy Experiments for Each Transition Group (the numbers are normalized by the magnitude of the aggregate welfare losses due to financial friction)

	(a) Loan per Business: $25,000			
	Partnership loan		Small business loan	
	Single owners	Partners	Single owners	Partners
Single owners	0	11.38	17.60	0.20
	-	(0.53)	(1.41)	(0.04)
Partners (G+L)	-0.64	4.66	5.17	0.86
	(0.03)	(0.18)	(0.64)	(0.08)
Group (L)	-0.64	-1.75	-0.05	-1.17
	(0.03)	(0.12)	(0.007)	(0.11)
Group (G)	0	6.41	5.21	2.03
	-	(0.27)	(0.65)	(0.17)
	(b) Loan per Business: $50,000			
	Partnership loan		Small business loan	
	Single owners	Partners	Single owners	Partners
Single owners	0	11.38	18.21	0.22
	-	(0.53)	(1.44)	(0.04)
Partners (G+L)	-0.64	4.70	5.43	0.89
	(0.03)	(0.18)	(0.67)	(0.09)
Group (L)	-0.64	-1.90	-0.05	-1.27
	(0.03)	(0.13)	(0.007)	(0.12)
Group (G)	0	6.60	5.48	2.16
	-	(0.28)	(0.68)	(0.17)

NOTE: The table presents welfare gains generated by two policies described in section 5. Group (L) indicates ex-partners who incur a loss due to each policy. Group (G) indicates ex-partners who benefit from each policy. A negative figure indicates the welfare *losses* from each policy. To highlight the extent of welfare improvement by policy, I normalize the gains/losses in both policies by the magnitude of the aggregate welfare losses due to financial friction. Bootstrap standard errors are in parentheses.

Figure 1: Kernel Density of Log Income

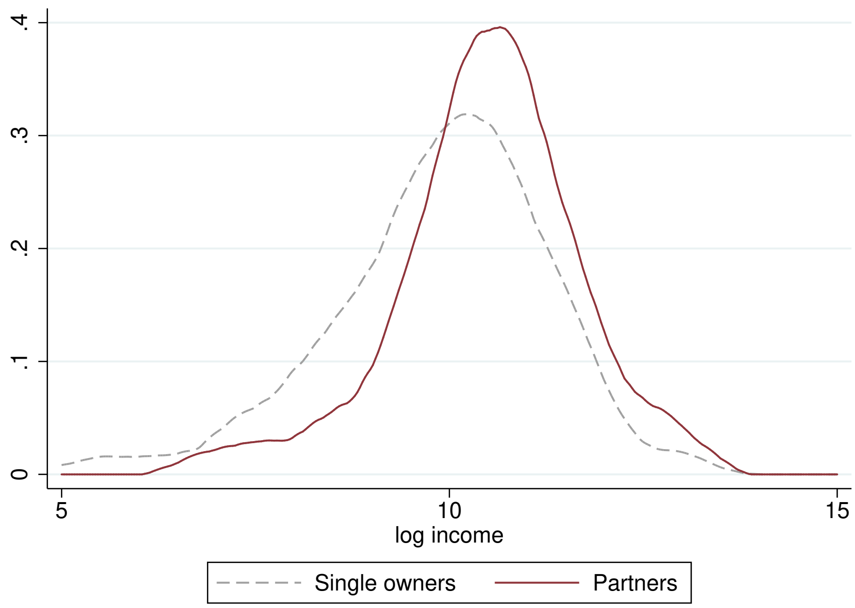


Figure 2: Kernel Density of Net Worth

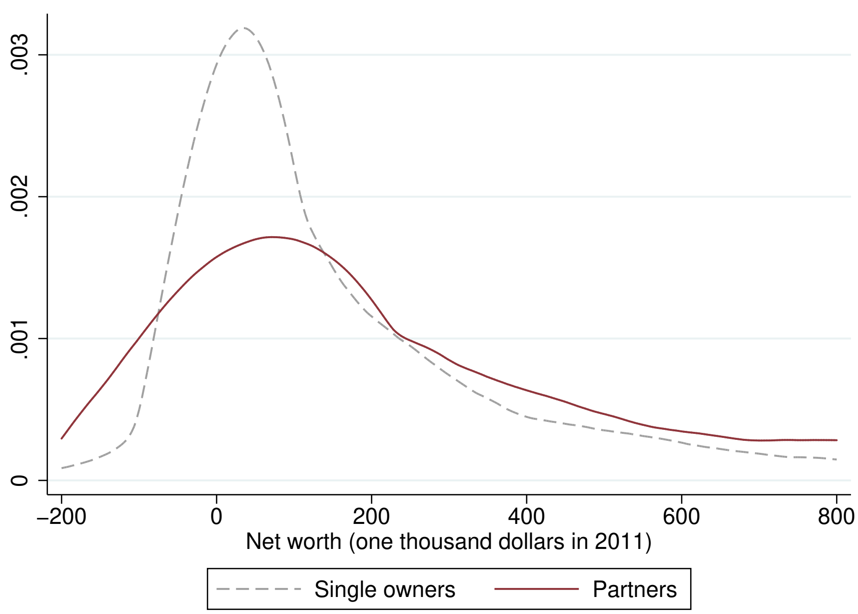
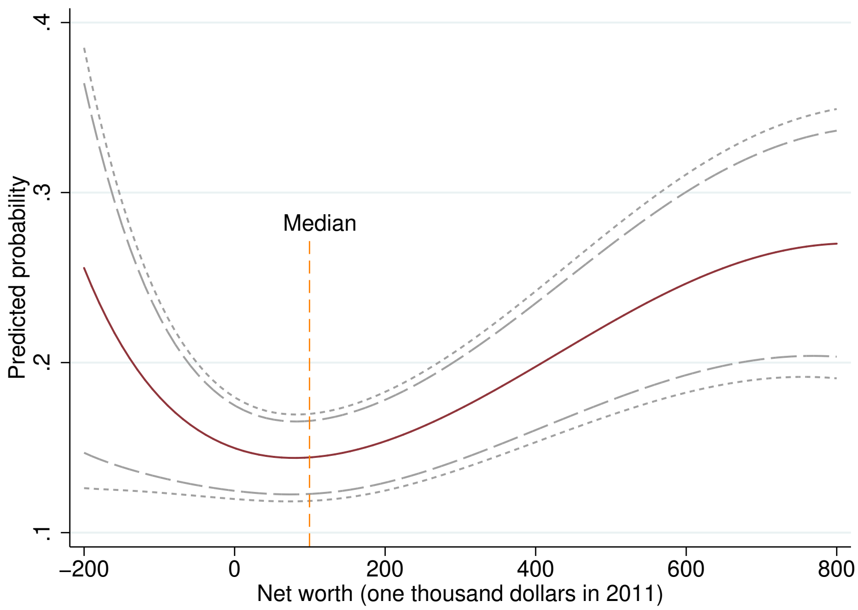
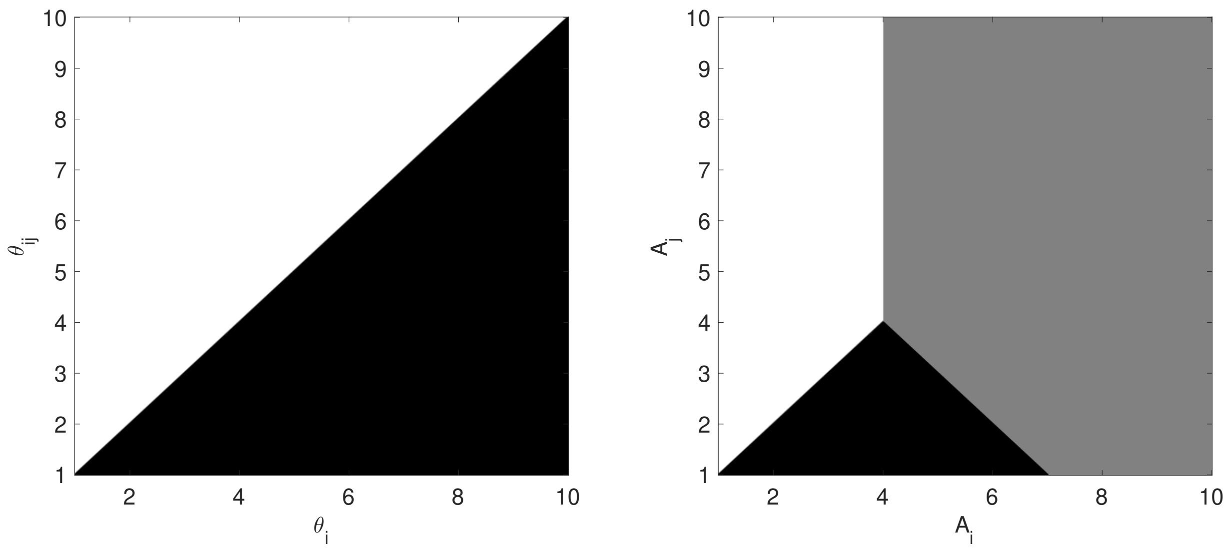


Figure 3: Net Worth and the Predicted Prob. of Becoming a Partner among Start-up Owners



NOTE: This figure depicts the predicted probability of becoming a partner among start-up owners with respect to net worth based on the estimates from the regression in Table 4. The median net worth is 96,753 USD. The long-dotted (short-dotted) gray lines represent the 90% (95%) bootstrap confidence interval.

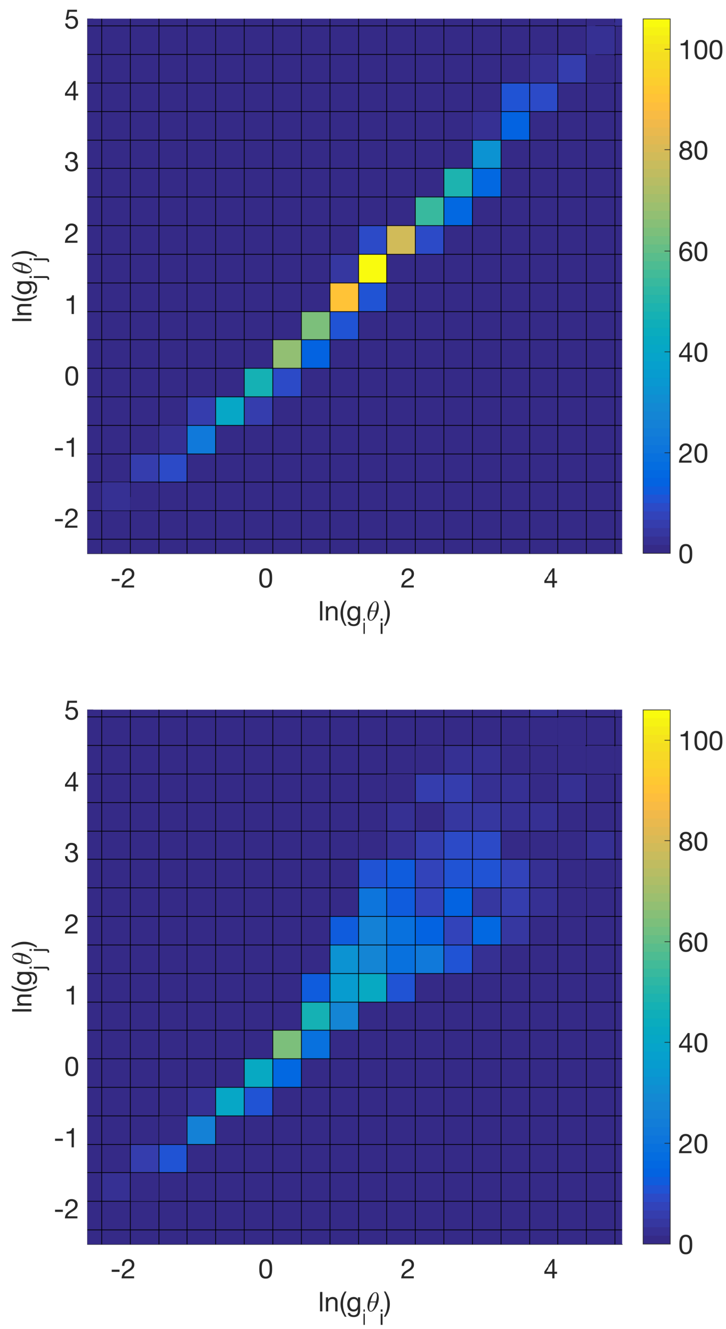
Figure 4: Graphical Representation of Proposition 2 and 3



(a) Proposition 2 (b) Proposition 3

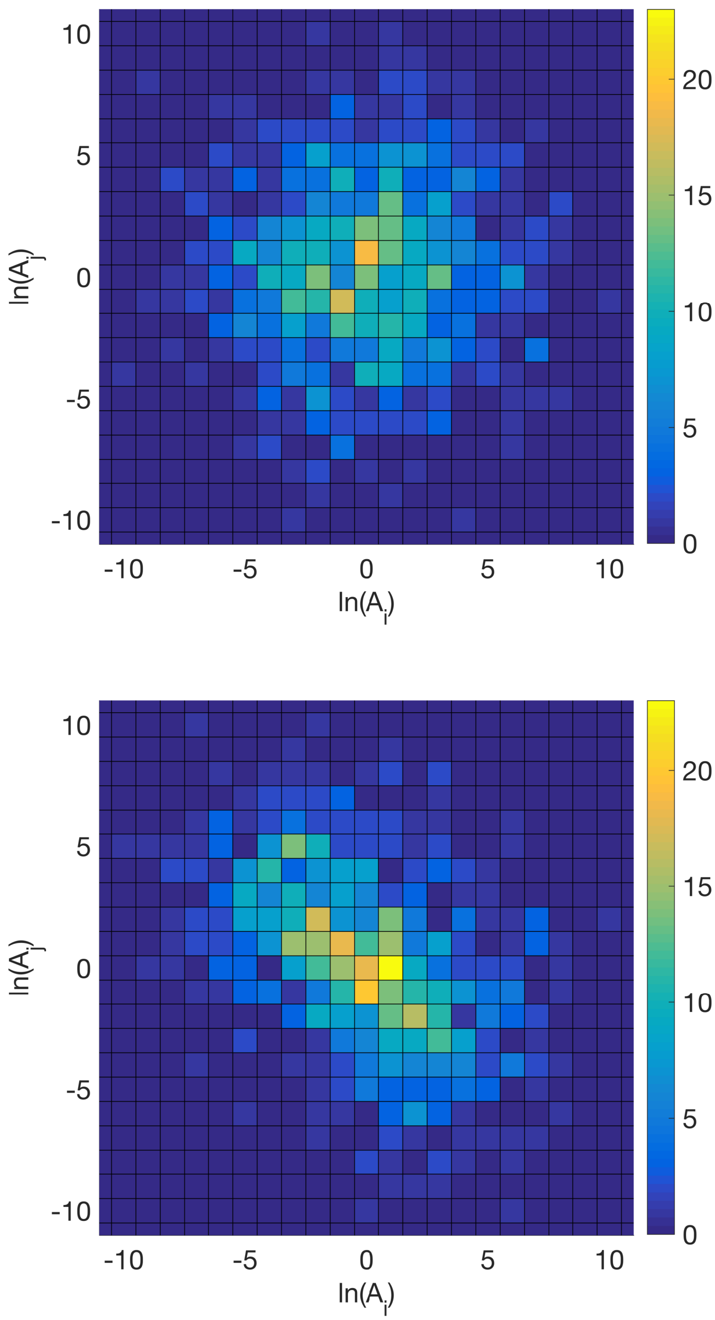
NOTE: These figures show whether agent i receives more value from the planner's problem (specified in section 3) than his value as a single owner. A point (θ_i, θ_{ij}) in Figure (a) represents a situation in which agent i with solo productivity θ_i is matched with agent j (with the same net worth) so that the partnership productivity is θ_{ij}. The white (black) color indicates agent i's value from the planner's problem is greater (smaller) than his value from single ownership. A point (A_i, A_j) in Figure (b) represents a situation in which agent i with net worth A_i is matched with agent j with net worth A_j (and with $\theta_i = \theta_{ij}$). In this example, agent i is not financially constrained as a single owner if $A_i \geq 4$, and is not financially constrained as a partner if $A_i + A_j \geq 8$. The white (black) color indicates agent i's value from the planner's problem is greater (smaller) than his value from single ownership. The gray color indicates agent i's value from the planner's problem is the same as his value from single ownership.

Figure 5: Financial Friction and Sorting: Ability



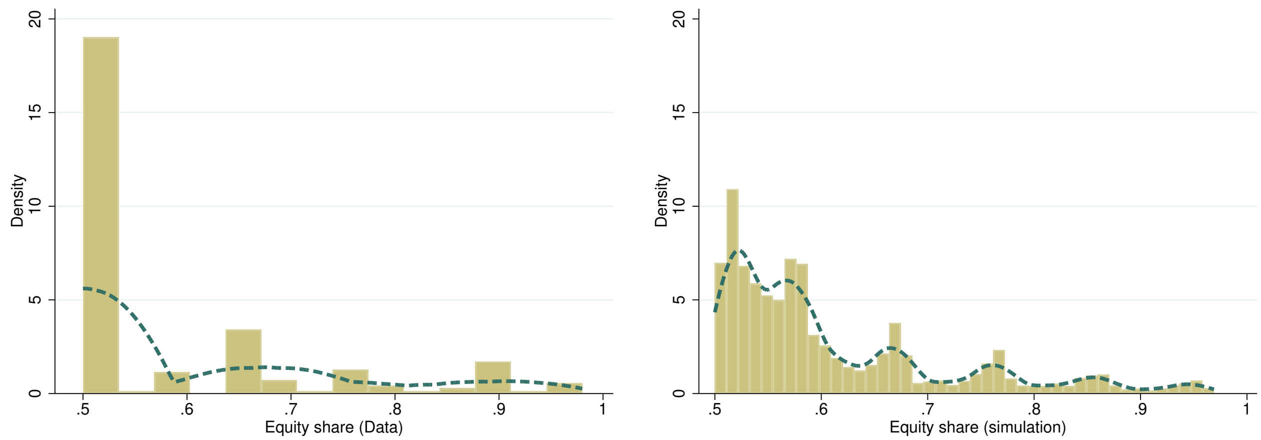
NOTE: The above two figures show the sorting patterns with respect to ability as a partner ($\ln(g_i\theta_i)$). The upper figure shows the sorting pattern without financial friction. The lower figure shows the sorting pattern when $\lambda = 1$. The correlation in the upper figure is 0.998 and the correlation in the lower figure is 0.904.

Figure 6: Financial Friction and Sorting: Wealth



NOTE: The above two figures show the sorting patterns with respect to the level of wealth ($\ln A_i$). The upper figure shows the sorting pattern without financial friction. The lower figure shows the sorting pattern when $\lambda = 1$. The correlation in the upper figure is 0.045 and the correlation in the lower figure is -0.46.

Figure 7: Histogram and Kernel Density (dotted line) for Equity Share

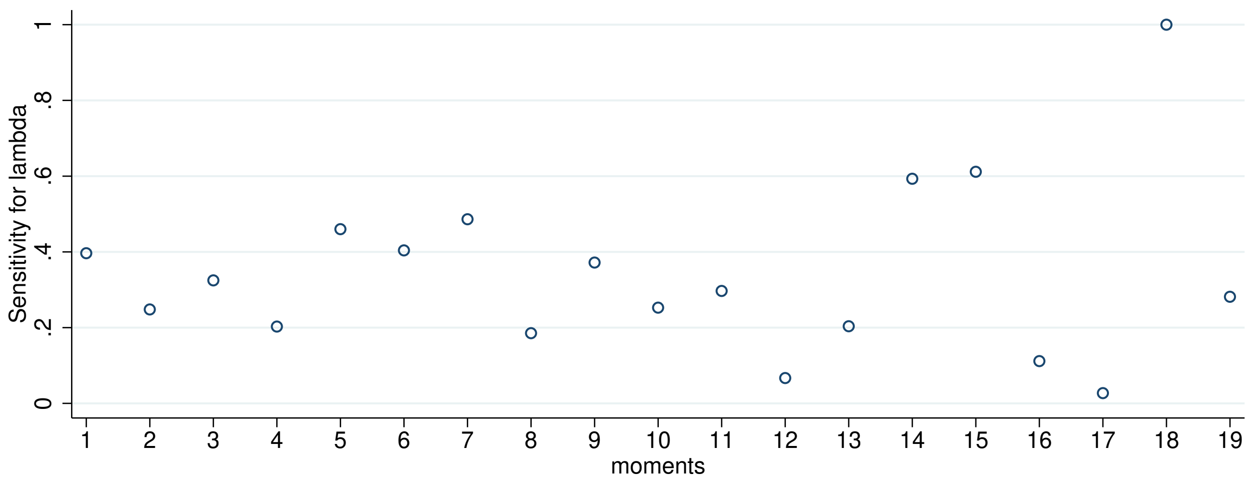


(a) Data

(b) Simulation

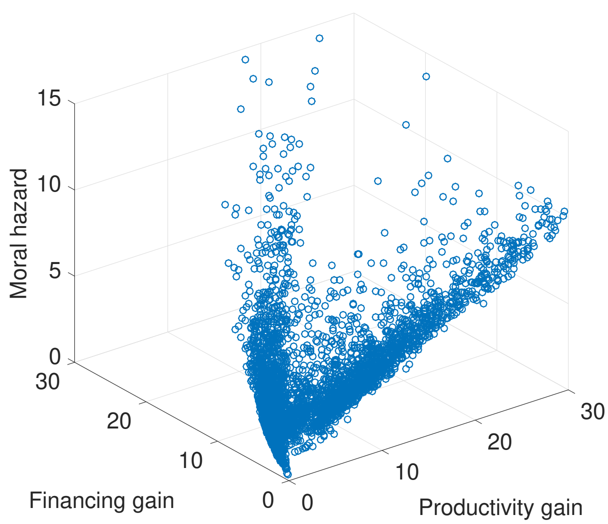
NOTE: This figure compares the histogram and kernel density for simulated equity shares with those from data. I simulate the model 100 times with the estimated parameters, and present all the equity shares from those simulations. For expositional purposes, I replace an equity share below one half with 1 minus the equity share.

Figure 8: Sensitivity Analysis for λ

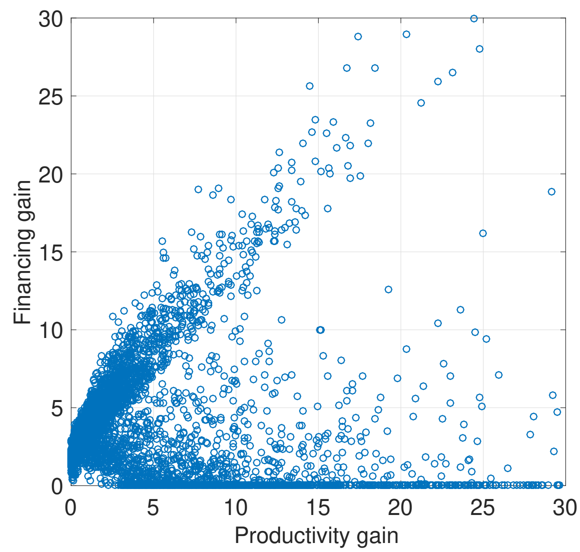


NOTE: The sensitivity of the parameter estimates with respect to the moment vector is a 15×19 matrix $\Lambda = -(\mathbf{D}'\mathbf{W}\mathbf{D})^{-1}\mathbf{D}'\mathbf{W}$. $\mathbf{W}$ is the 19×19 weighting matrix, and $\mathbf{D} = \frac{\partial \mathbf{m}_{sim}(\psi)}{\partial \psi}|_{\psi=\hat{\psi}}$ is the 19×15 derivative matrix of the moment vector with respect to the model parameters evaluated at the estimates. Note the number of moments and the number of model parameters are 19 and 15, respectively. Given the scales of the moments are not always comparable, I multiply the mth column of Λ by the standard deviation of the mth moment following Gayle and Shephard [2019]. The sensitivity of the borrowing-constraint parameter λ is the row of the sensitivity matrix corresponding to λ. The presented circle represents the absolute value of each element in the sensitivity of λ with respect to the corresponding moment, normalized by the highest absolute value among those elements. The number for a moment is identical to the one in Table 8. The 18th moment reflects the information about the skewness of partner income conditional on net worth.

Figure 9: Scatter Plots for Benefits and Costs of Partnerships



(a) Three dimensional representation



(b) Two dimensional representation

NOTE: Figure (a) depicts the scatter plot for productivity gains (Ω_1), financing gains (Ω_2), and the cost of moral hazard (Ω_3) in equation (9) for partnerships. With the estimated model parameters, I simulate the economy 50 times and present the results for all the partnerships from the simulations. For illustration purposes, I present $\{\Omega_1, \Omega_2, -\Omega_3\}$ for each partnership. Figure (b) is the two-dimensional representation of Figure (a) with respect to productivity and financing gains.

Appendix

A Computational Algorithm

To find an equilibrium, I apply a transfer-adjustment process proposed by Crawford and Knoer [1981]. The process starts with agents on one side, say, Side 1, making an offer to an agent on Side 2 as follows:

STEP1

- An agent (say, agent i) on Side 1 calculates the amount of equity transfer to an agent (say, agent j) on Side 2, which evenly compensates the outside option value for the agent on Side 2. Let $\tau_j^i(0)$ be such value.

STEP2

- An agent on Side 1 finds the agent on Side 2 with whom he receives the highest value given the the amount of equity transfers calculated at STEP 1. In other words, agent i on Side 1 solves $\max_j \left\{V_{ij}(\theta_{ij}, A_{ij}, 1-\tau_j^i(0))\right\}$. If the maximized value is less than the outside option of agent i, agent i does not make an offer, and chooses to be a single owner. If the maximized value is greater than the outside option, agent i makes an equity offer $\tau_{j^*}^i(0)$ to agent j^* on Side 2, where $j^* = \arg\max_j \left\{V_{ij}(\theta_{ij}, A_{ij}, 1-\tau_j^i(0))\right\}$.

STEP3

- An agent on Side 2 who receives one or more offers rejects all but his favorite, which he tentatively accepts.

STEP4

- Offers not rejected in the previous round remain in force. If agent j on Side 2 rejects an offer from agent i on Side 1 in the previous round, agent i updates the equity offer to agent j as $\tau_j^i(R) = \tau_j^i(R-1) + d$, where d is a constant. If not, $\tau_j^i(R) = \tau_j^i(R-1)$. Note R represents the round of offers agents on Side 1 make to agents on Side 2. Rejected agents on Side 1 continue to make offers to their favorite agents on Side 2, taking into account their current permitted transfers.

STEP5

- The process stops when no rejection occurs.

Crawford and Knoer [1981] show this process generates a stable matching and terminates with a finite round. The outcome of this algorithm depends on the incremental equity offer (d). The termination time increases as d becomes smaller. To make computation feasible, I assume $d = 0.1$, meaning the equity offer increases by 10%.

B Robustness Checks

The matching function (5) is assumed to be symmetric in the sense that the contribution of each partner's productivity to his partnership is the same. To check the extent to which the main results in the article are sensitive to this symmetry assumption, I consider the following modified matching function:

$$\theta_{ij} = \hat{\theta}_h^{\xi}\hat{\theta}_l^{1-\xi}, \quad \text{where} \quad \hat{\theta}_h = \max\{g_i\theta_i, g_j\theta_j\}, \quad \hat{\theta}_l = \min\{g_i\theta_i, g_j\theta_j\}, \quad \xi \geq 0.5.$$

In this matching function, the more productive partner's contribution to the partnership productivity becomes greater as ξ increases. The specification in the article is a special case of this matching function when $\xi = 0.5$.

I conduct two robustness checks with respect to different values of ξ. Specifically, I *re-estimate* the model parameters by assuming (1) $\xi = 0.6$ and (2) $\xi = 0.7$. With the newly estimated parameters, I check the extent to which the main results in the article are sensitive to the assumption on ξ.

Table 16 shows the decomposition results with the estimates from different values of ξ. As ξ increases, the relative value of financing gains over productivity gains increases (Panel (a)). Other decomposition results (e.g., the total gains, gains/losses conditional on net worth) are very similar across the three specifications.

As long as ξ is less than 1, the partnership productivity depends on both partners' productivity as a partner. When a highly productive individual searches for his partner in order to seek financial help, he will consider not only his partner's wealth, but also his partner's productivity, because if he is matched with a partner whose productivity is very low, the partnership productivity will decrease too much even though his partner has a large amount of wealth. This trade-off − the trade-off between the partner's wealth and partner's productivity − will be a lesser concern as ξ increases, because the productivity loss from being matched with a partner with a lower productivity becomes smaller as ξ increases. As a consequence, as ξ increases, high-productivity individuals form partnership firms that generate greater financing gains in a stable matching.

To summarize, as ξ increases, the relative importance of the financing gain increases. The change in the magnitude is, however, relatively small (from 39.12 to 42.37) when ξ increases from 0.5 to 0.7. Other results such as the total gains and gains/losses conditional on net worth do not change much when ξ increases from 0.5 to 0.7.

Table 16: Decomposing Benefits and Costs of Partnerships with Different ξ

	$\xi = 0.5$	$\xi = 0.6$	$\xi = 0.7$
Total gains relative to the aggregate partnership income (%)	35.59	35.44	35.22
Panel (a). All partners			
Productivity gain (%)	60.88	59.35	57.63
Financing gain (%)	39.12	40.65	42.37
Moral hazard (%)	-42.29	-42.43	-43.71
Panel (b). Partners $<$ 25th wealth			
Productivity gain (%)	7.30	7.20	7.31
Financing gain (%)	92.70	92.80	92.96
Moral hazard (%)	-37.20	-38.87	-42.42
Panel (c). Partners $\geq$ 25th wealth			
Productivity gain (%)	98.85	98.80	98.83
Financing gain (%)	1.15	1.20	1.17
Moral hazard (%)	-45.90	-45.13	-44.77

NOTE: Panel (a) shows the decomposition of benefits and costs from partnerships for all partners. The aggregate gains for all partners are normalized as 100. Panels (b) and (c) show the decomposition of benefits and costs from partnerships below and above the 25th percentile of the wealth distribution, respectively. The aggregate gains for each group are normalized as 100.

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